DC1: CFD techniques for IBRA-type discretizations.

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Design for *IGA*-type discretization workflows

1st Technical Workshop

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Summary

Summary

Fitted VS Un-fitted methods

The Shifted Boundary Method (SBM)

SBM in IGA & *Results*:
- External vs Optimal boundaries
- Convergence studies
- Condition Number

SBM vs Trimming & *Results:*
- Convergence studies

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Introduction

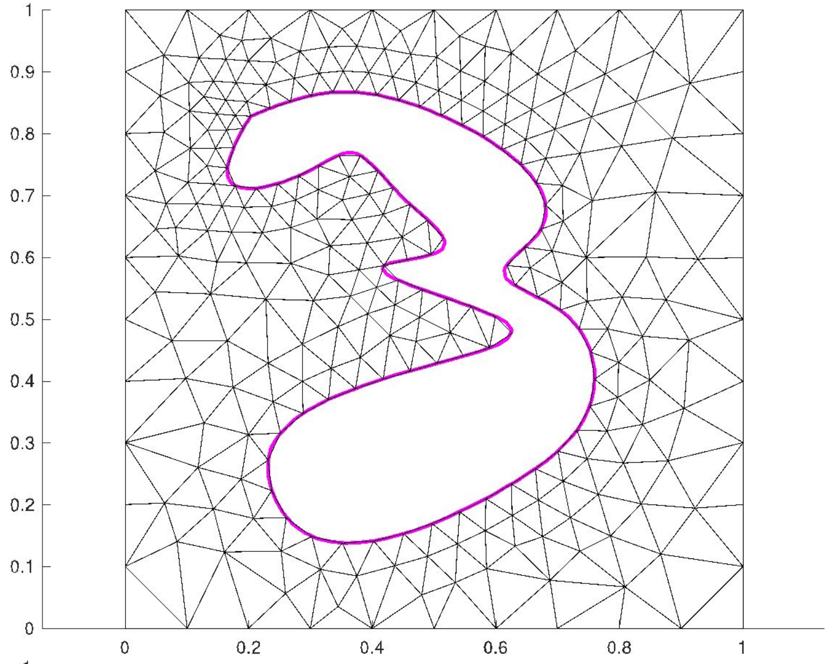
Fitted VS Un-fitted methods

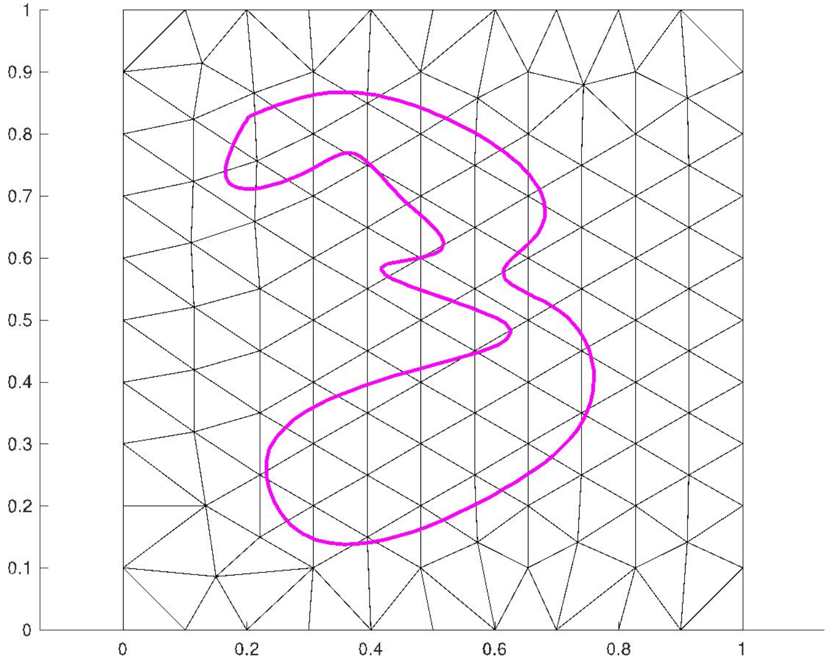
Un-fitted Methods

Were born as an alternative to the classical body-fitted approach

To represent:
- immersed objects
- external boundaries

The domain is meshed independently from the embedded boundary.





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Introduction

Fitted VS Un-fitted methods

Un-fitted Methods

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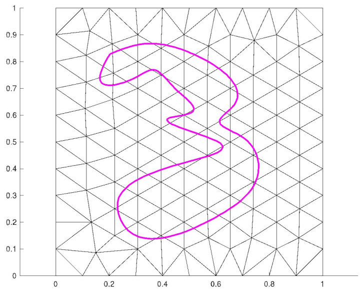

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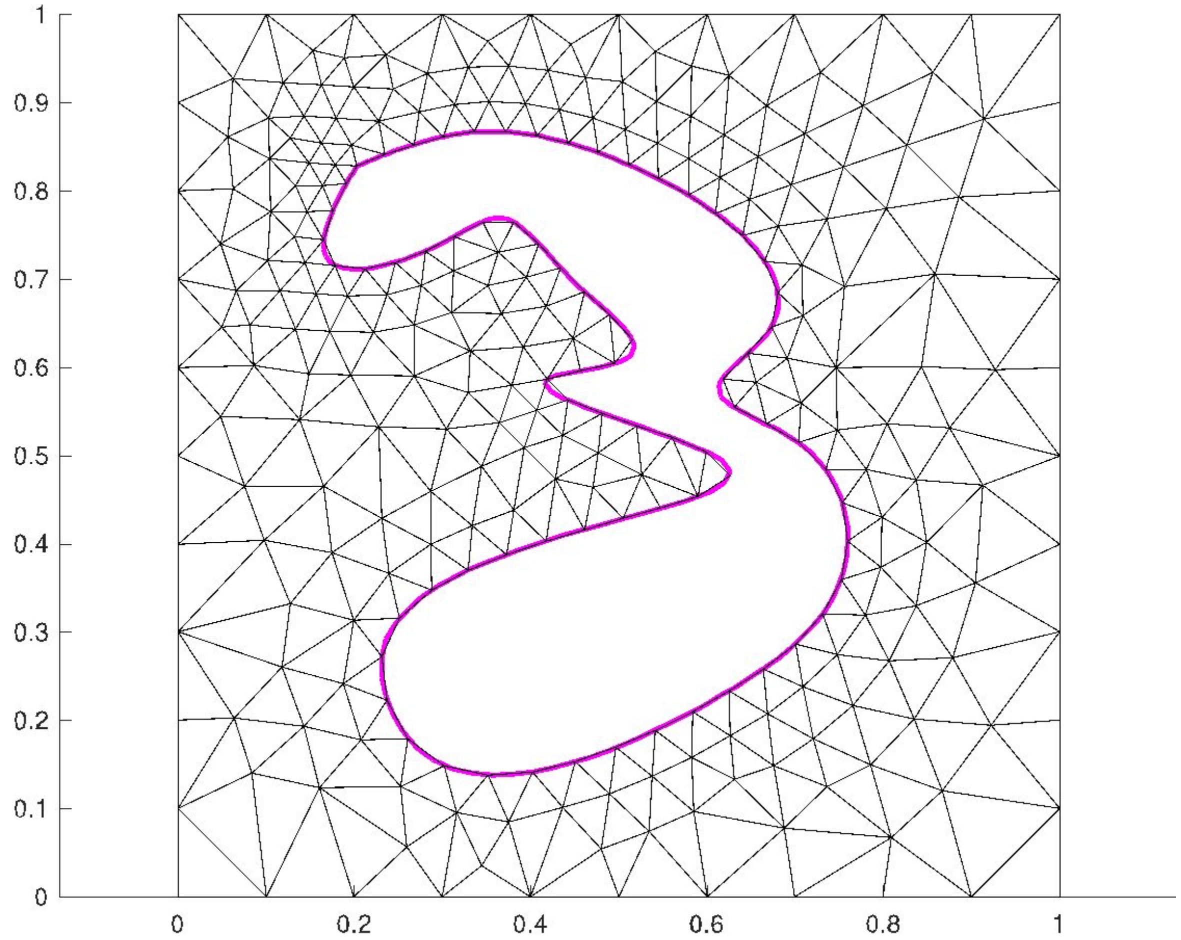

Body-fitted approach

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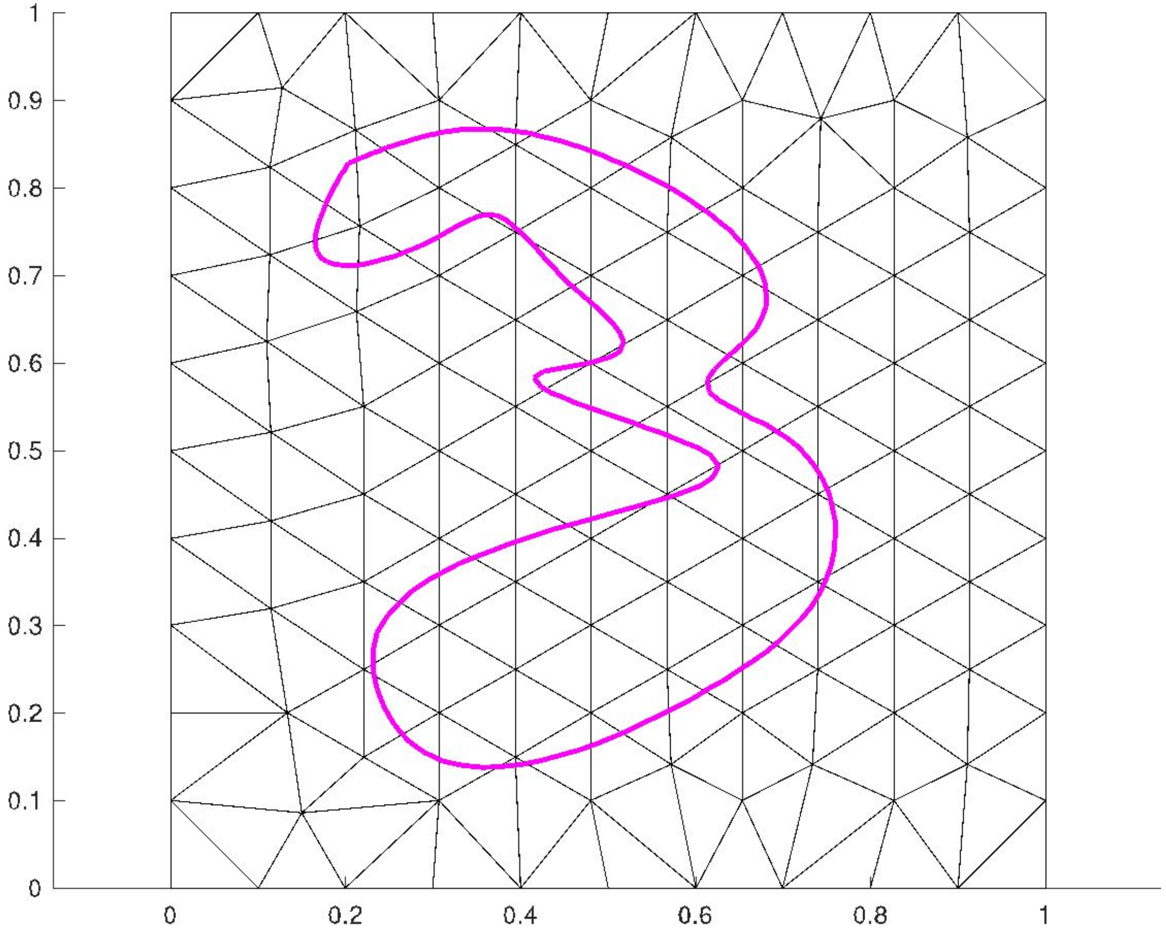

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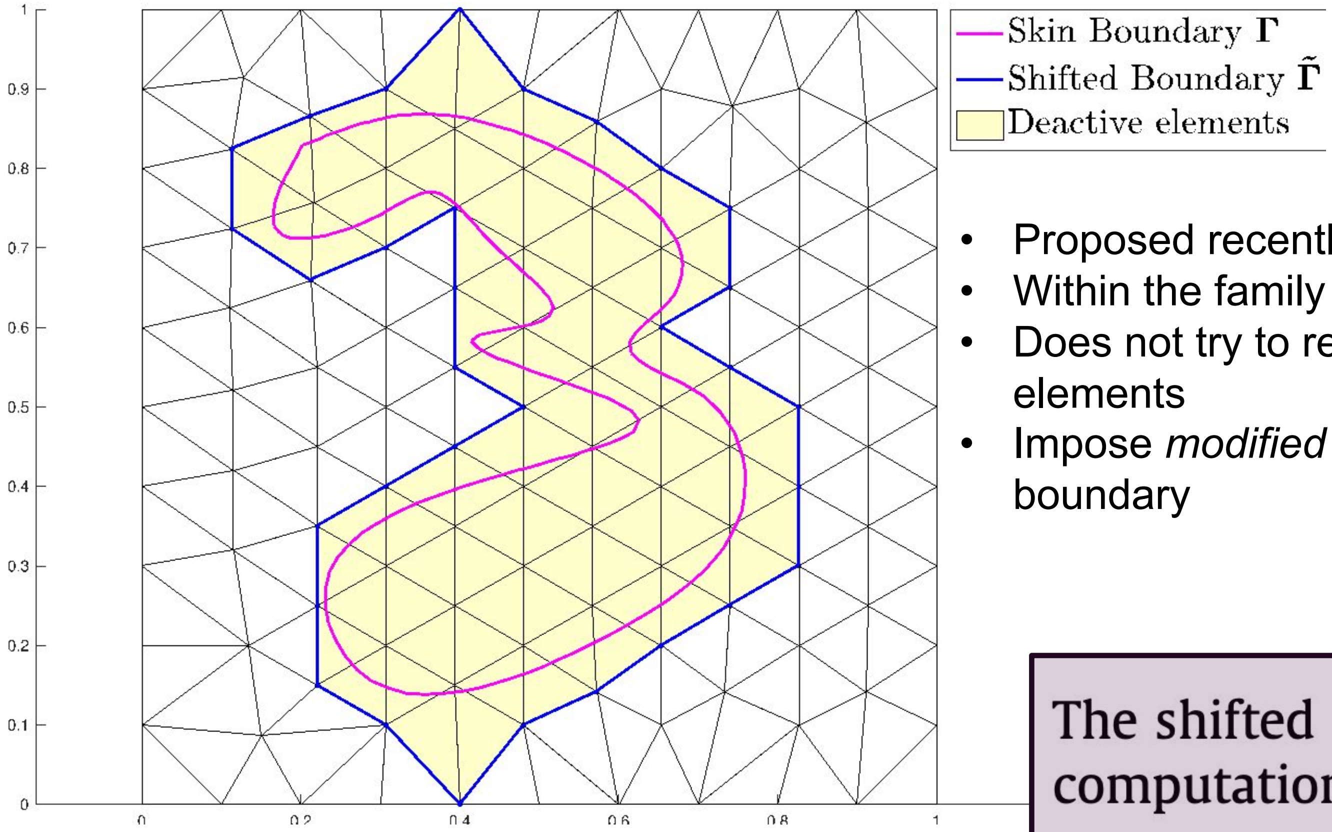
The Shifted Boundary Method (SBM)



- Proposed recently by Professor Scovazzi
- Within the family of *approximate boundary methods*
- Does not try to reconstruct the embedded interface in the cut elements
- Impose *modified* Dirichlet boundary conditions at the shifted boundary

The shifted boundary method for embedded domain computations. Part I: Poisson and Stokes problems

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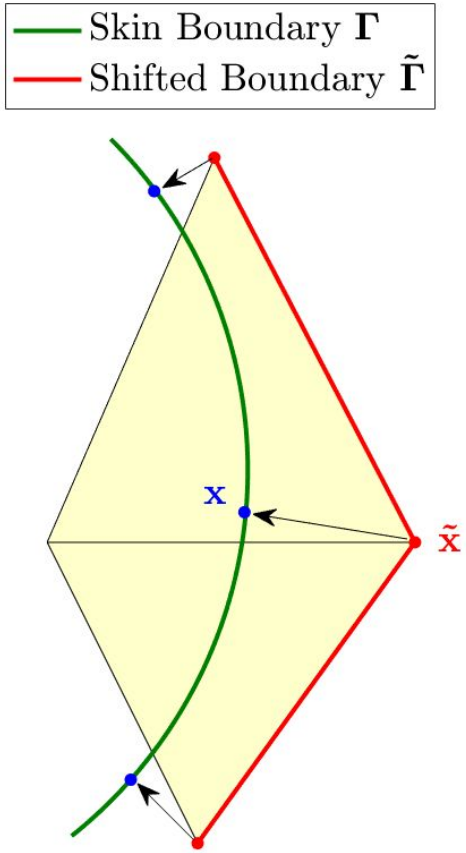
The Shifted Boundary Method (SBM)



- Impose *modified* Dirichlet boundary conditions at the shifted boundary

In order to preserve a 2nd order convergence we have to take into account the *gradient term*:

Taylor expansion:

$$u(\boldsymbol{x}) = u(\tilde{\boldsymbol{x}}) + \nabla u|_{\tilde{\boldsymbol{x}}} \cdot (\boldsymbol{x} - \tilde{\boldsymbol{x}}) + o(|(\boldsymbol{x} - \tilde{\boldsymbol{x}})^2|)$$

$$u(\tilde{\boldsymbol{x}}) \approx u(\boldsymbol{x}) - \nabla u|_{\tilde{\boldsymbol{x}}} \cdot (\boldsymbol{x} - \tilde{\boldsymbol{x}})$$

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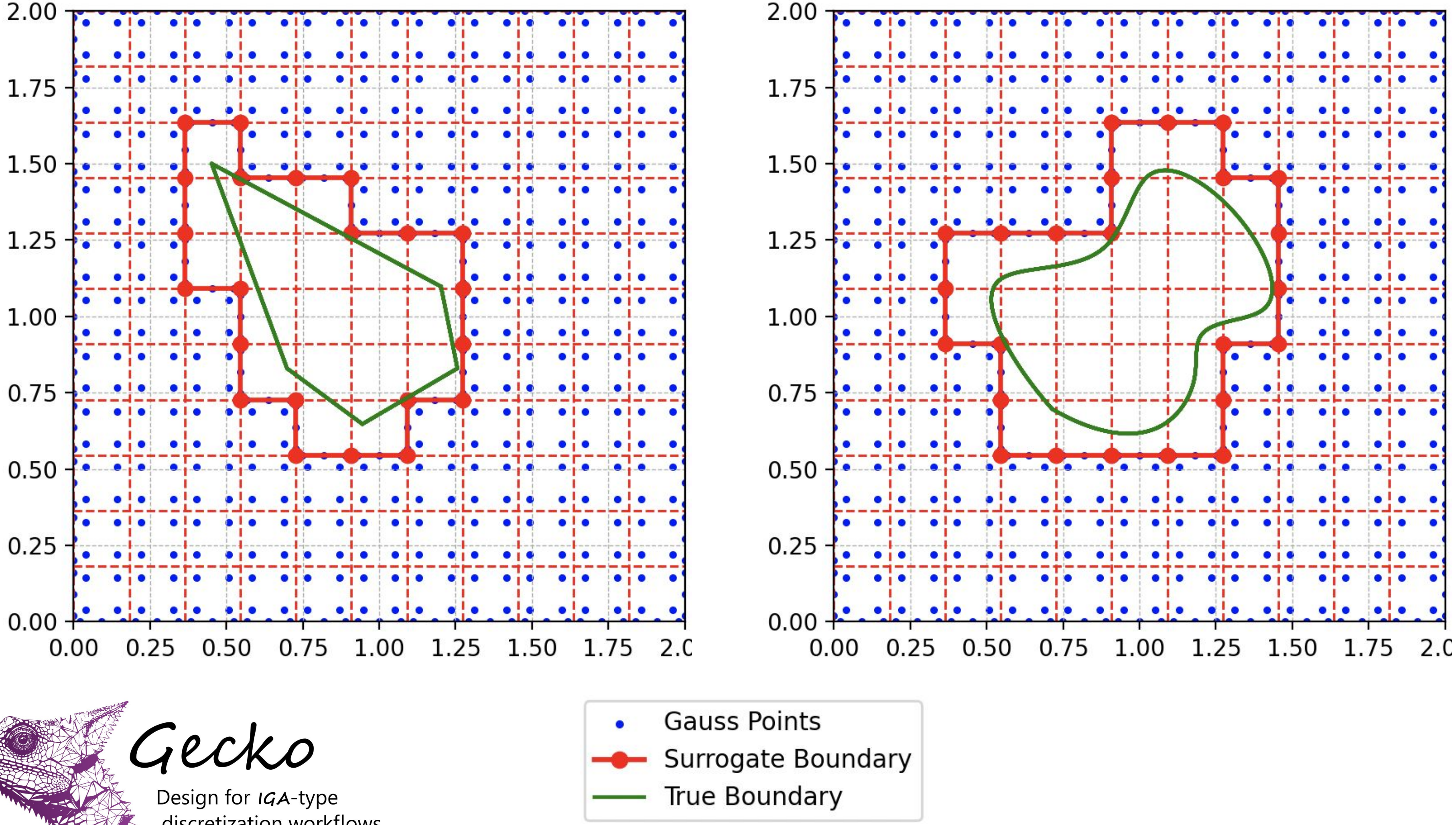
SBM in IGA



For the moment everything is considered in the **parameter space**.

In the future, the true boundary will be considered in the physical and will need to be mapped to the parameter space.

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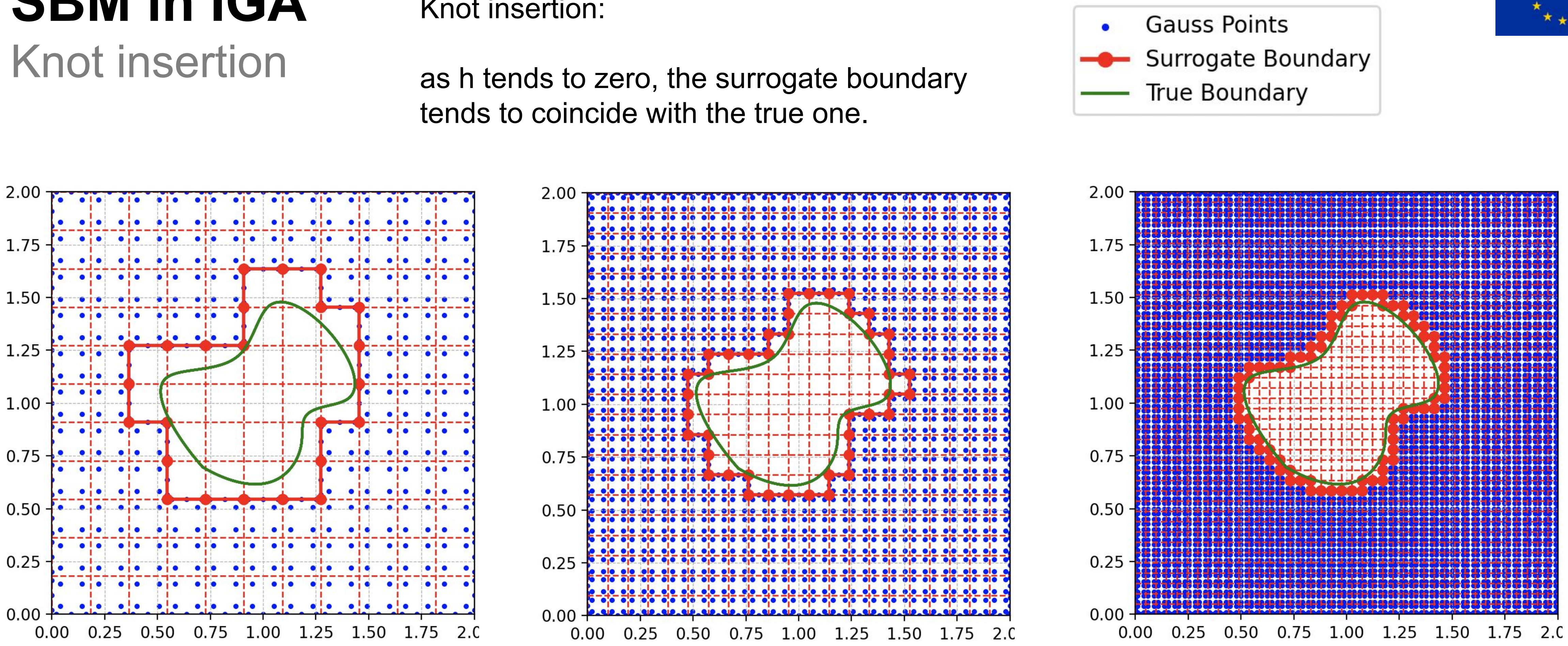
Knot insertion

Knot insertion:

as h tends to zero, the surrogate boundary tends to coincide with the true one.



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Degree elevation

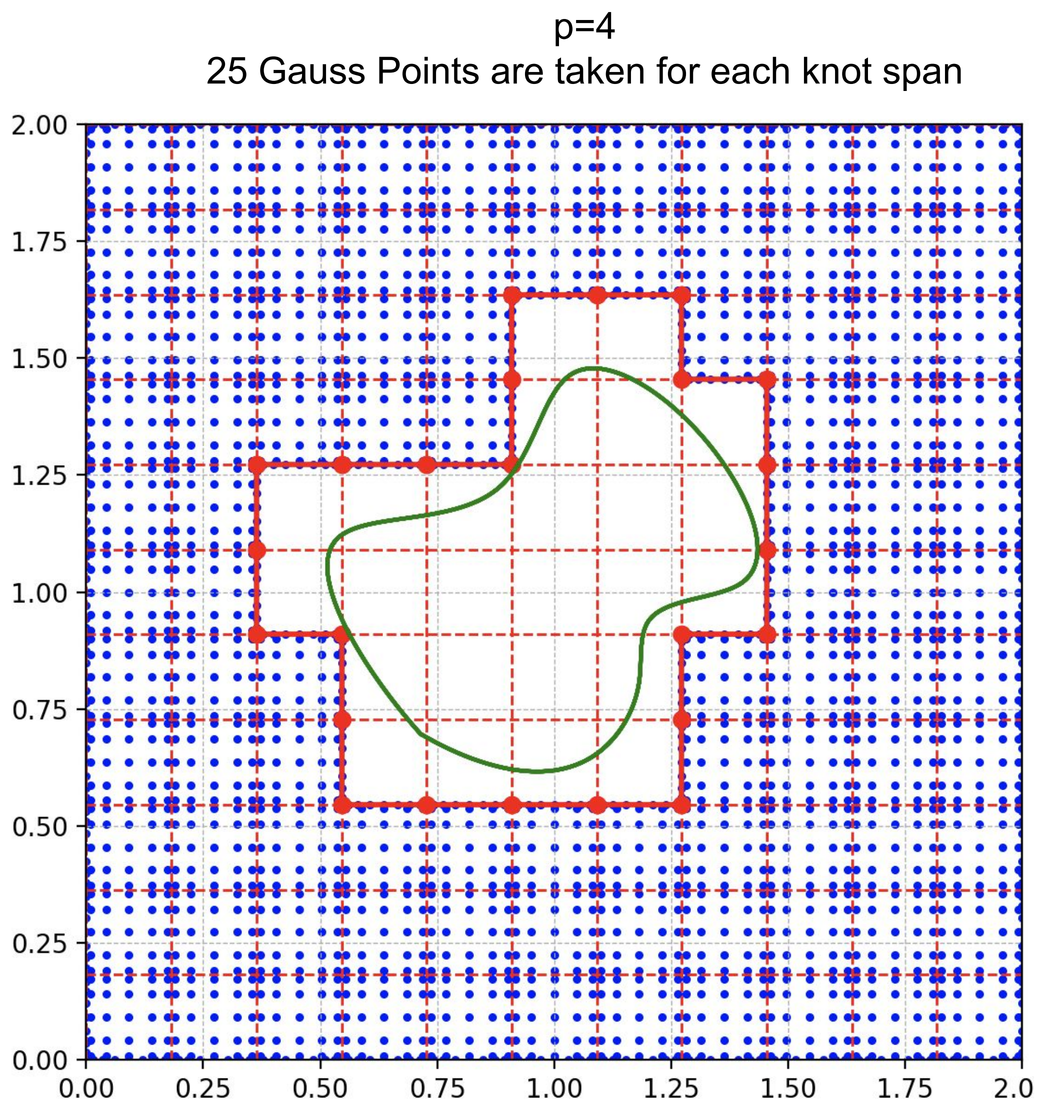
Degree elevation is also possible. The Taylor expansion between the true and surrogate boundary must be up to the p+1 order.

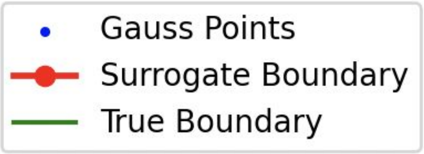
Example:

using quadratic IGA basis function (p=2) we will use the following Taylor expansion to impose the surrogate BCs:

$$u(\mathbf{x}) = u(\hat{\mathbf{x}}) + \nabla u|_{\hat{\mathbf{x}}} \cdot (\mathbf{x} - \hat{\mathbf{x}}) + \frac{1}{2}(\mathbf{x} - \hat{\mathbf{x}})^T \cdot \mathbf{H}u|_{\hat{\mathbf{x}}} \cdot (\mathbf{x} - \hat{\mathbf{x}})$$

$\mathbf{H}u|_{\hat{\mathbf{x}}}$ *is the Hessian matrix evaluated at the surrogate location.*





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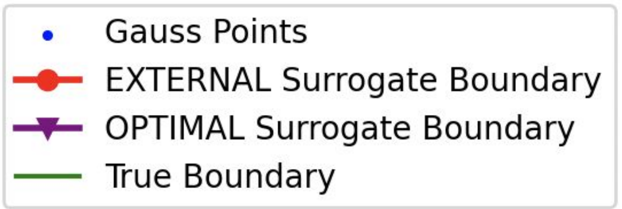
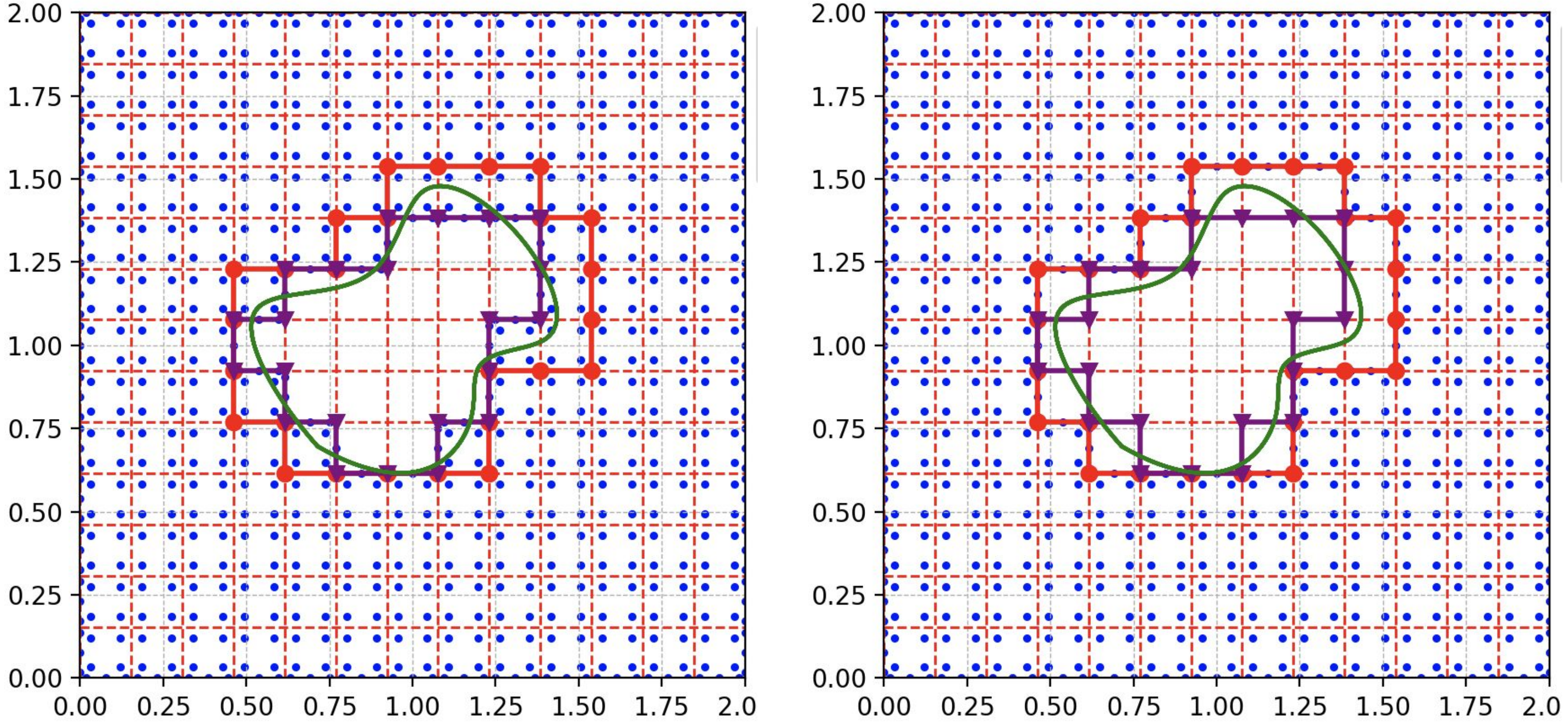
Results: External and Optimal Boundary

We can enhance the Shifted Boundary Method by considering the ***optimal*** boundary instead of the ***external*** one.

↓

In this way the Taylor Expansion should cover a distance which is at most *h/2*, instead of *h*.





Using the optimal boundary

Using the external boundary

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Results: Convergence Studies

All the convergence studies are performed on a 2D Poisson problem with Dirichlet BCs. We have an internal hole defined through an SB method. Therefore:

- **External body-fitted Dirichlet BCs.**
- **Internal SB Dirichlet conditions.**

Poisson problem:

$$
\begin{aligned}
-\Delta u &= f && \text{on } \Omega \\
u &= u_D && \text{on } \Gamma_D = \partial\Omega
\end{aligned}
$$

Manufactured solution:

$$u(x, y) = \sin(x)\sinh(y)$$

Penalty-Free weak formulation for imposing Dirichlet BCs:

$$a_h^k(u_h, w_h) = (\nabla u_h, \nabla w_h)_{\tilde{\Omega}_h} - \langle \nabla u_h \cdot \tilde{\boldsymbol{n}}, w_h \rangle_{\tilde{\Gamma}_h} + \langle \mathsf{S}_{\boldsymbol{\delta}}^k u_h, \nabla w_h \cdot \tilde{\boldsymbol{n}} \rangle_{\tilde{\Gamma}_h}$$

$$l_h(w_h) = (f, w_h)_{\tilde{\Omega}_h} + \langle \bar{u}_D, \nabla w_h \cdot \tilde{\boldsymbol{n}} \rangle_{\tilde{\Gamma}_h} .$$

A penalty-free Shifted Boundary Method of arbitrary order

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[b] *Université de Franche-Comté, CNRS, LmB, F-25000 Besançon, France*

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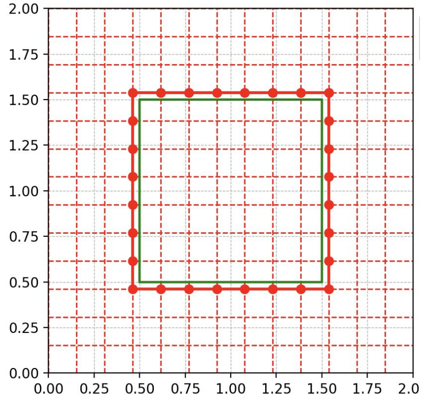
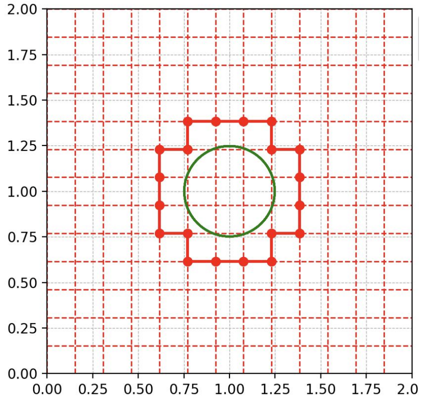
Results: External Boundary

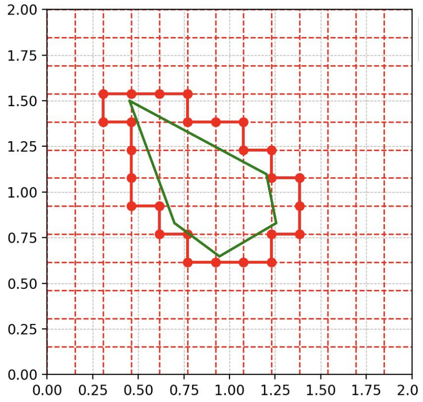
Comparison p = 1, 2, 3 with **EXTERNAL** surrogate boundary with three shapes:



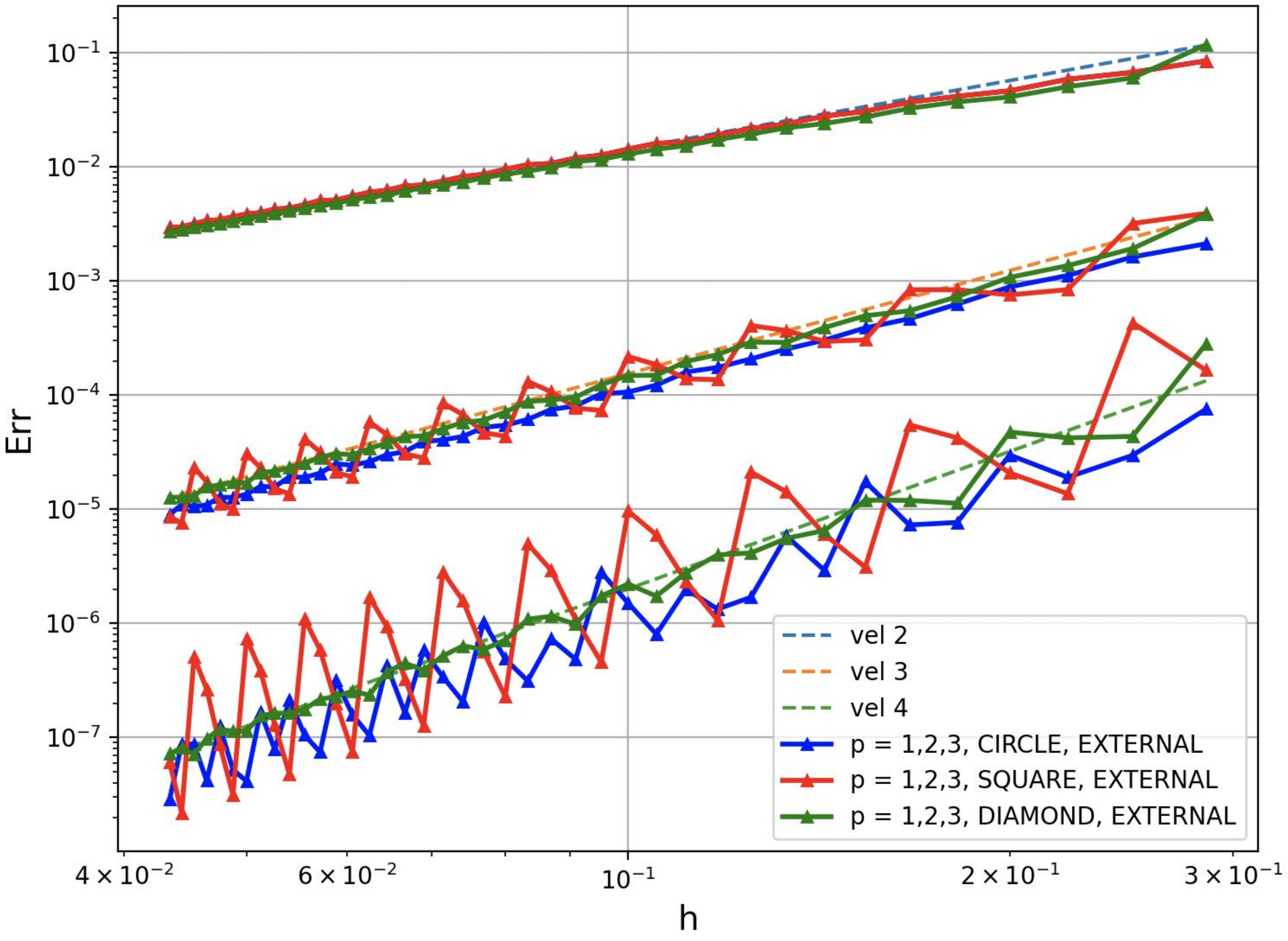
SQUARE



CIRCLE



DIAMOND



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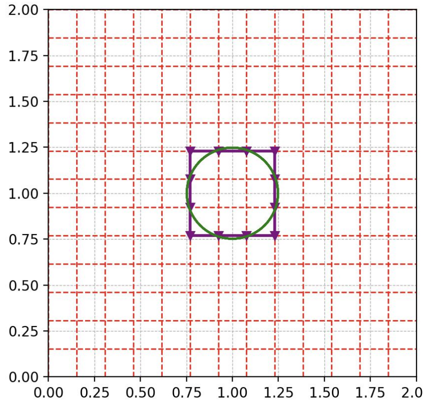
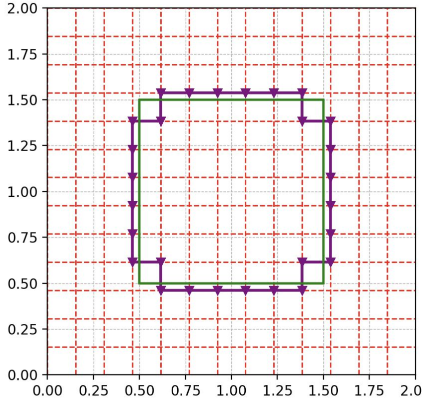
Results: Optimal Boundary

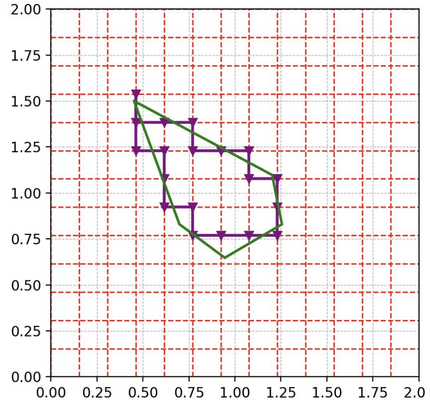
Comparison p = 1, 2, 3, 4 with **OPTIMAL** surrogate boundary with three shapes:



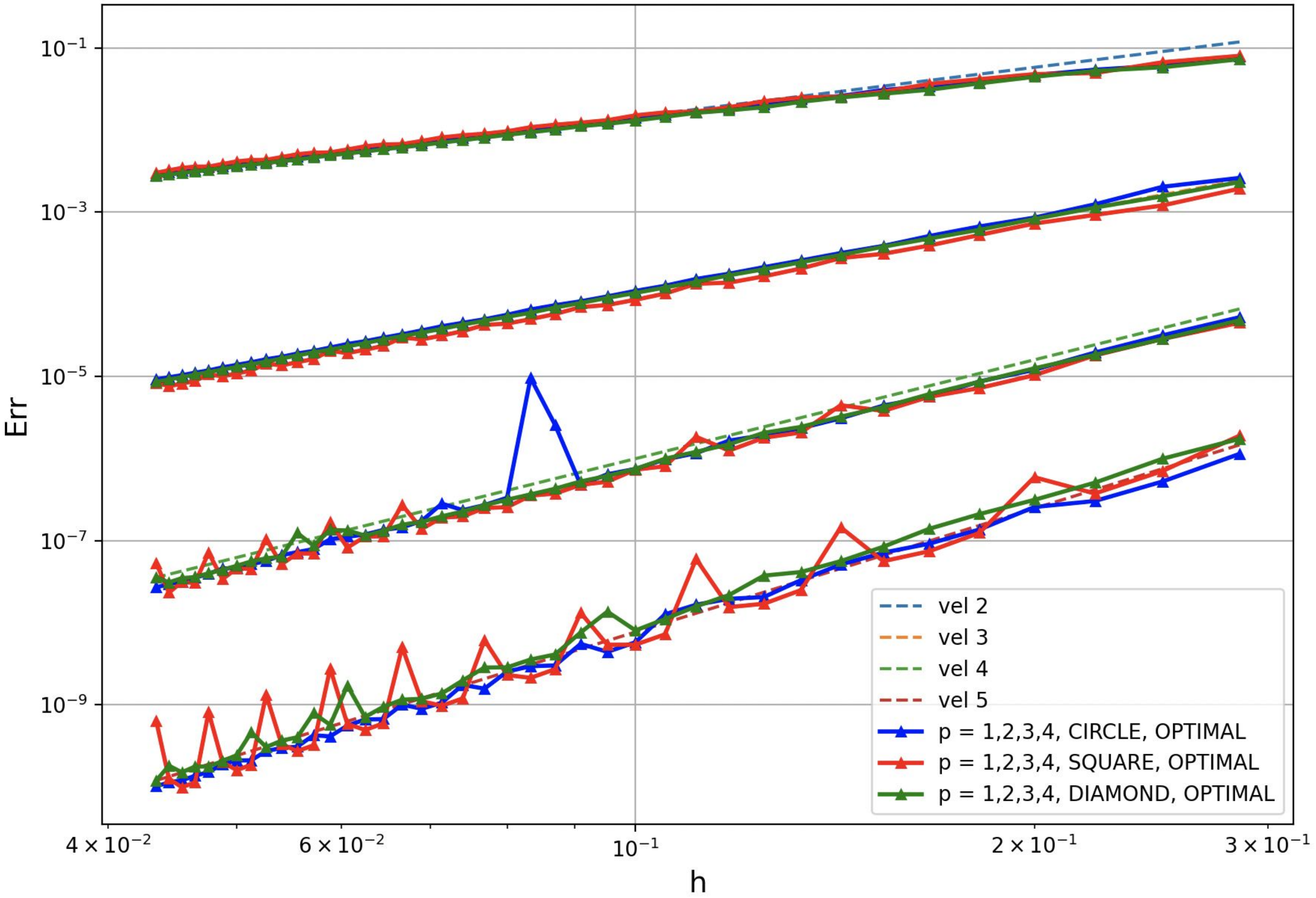
CIRCLE



SQUARE



DIAMOND



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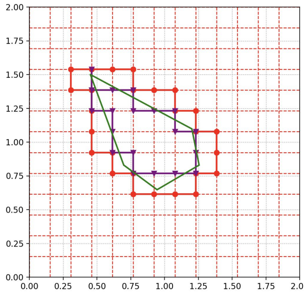
SBM in IGA

Results: Body-Fitted vs External & Optimal

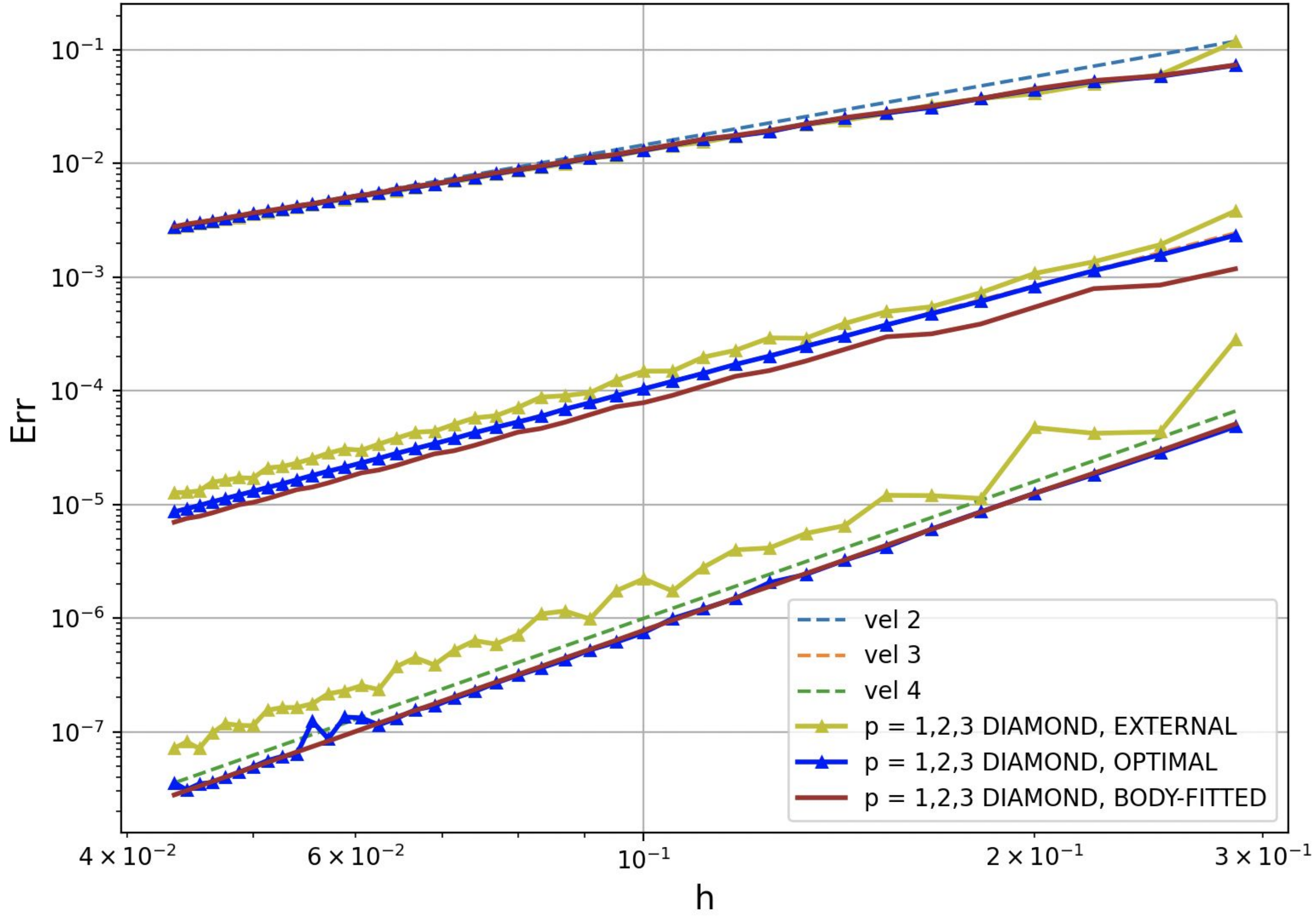
Comparison using p = 1, 2, 3 of the DIAMOND case (which has not any particular symmetry).

In the following cases:

- **Body-Fitted** approach along the surrogate boundary
- **External** Surrogate Boundary
- **Optimal** Surrogate Boundary



DIAMOND



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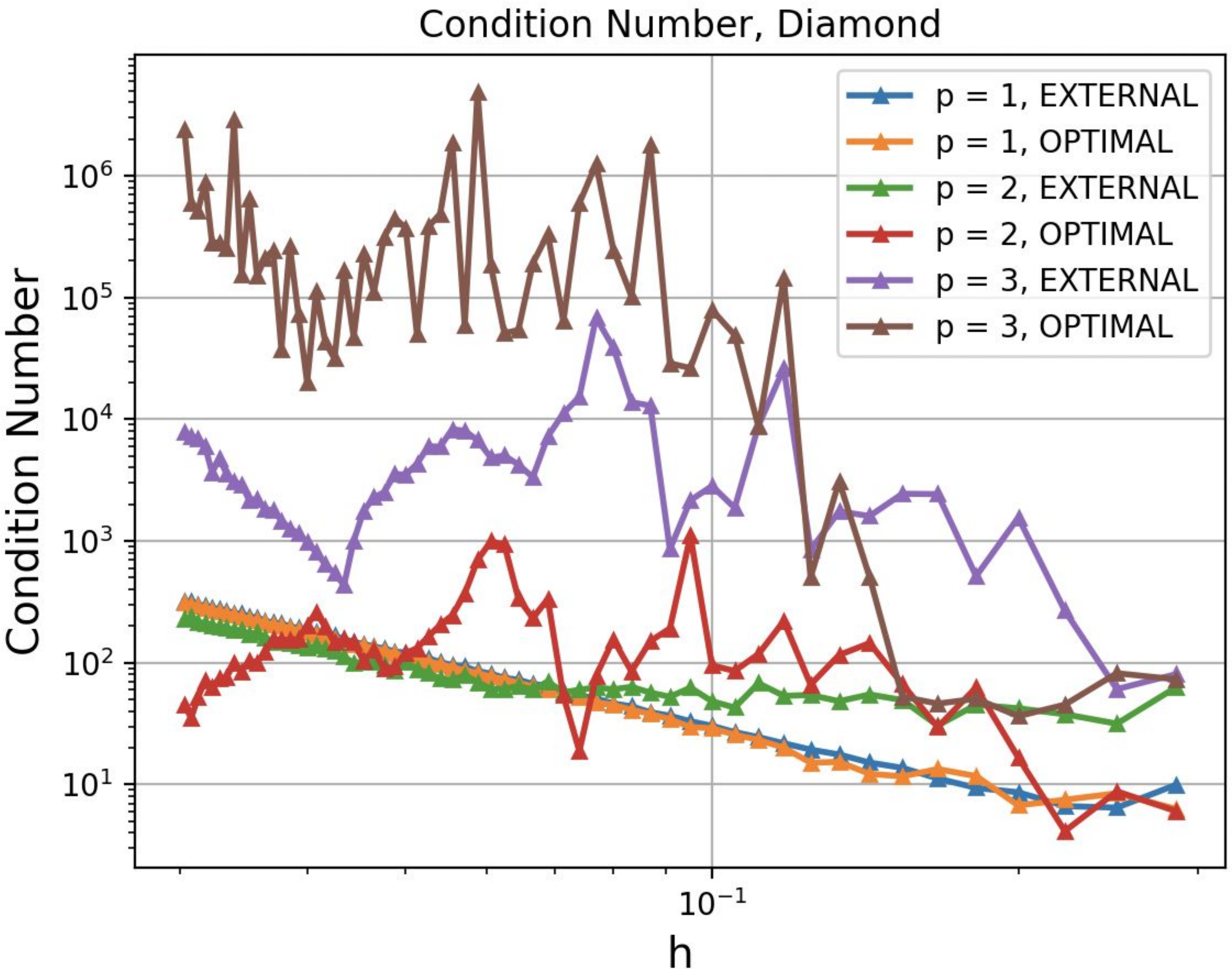
Results: Condition Number

The condition number is a measure of the matrix's sensitivity to numerical errors and its stability in solving the linear system.

$$\kappa(A) = \frac{\lambda_{\max}}{\lambda_{\min}}$$

Cut-FEM approaches suffer the ***small cut-cell problem*** which is caused by arbitrary small cut elements (huge condition numbers).

SBM avoids integrating the cut elements.



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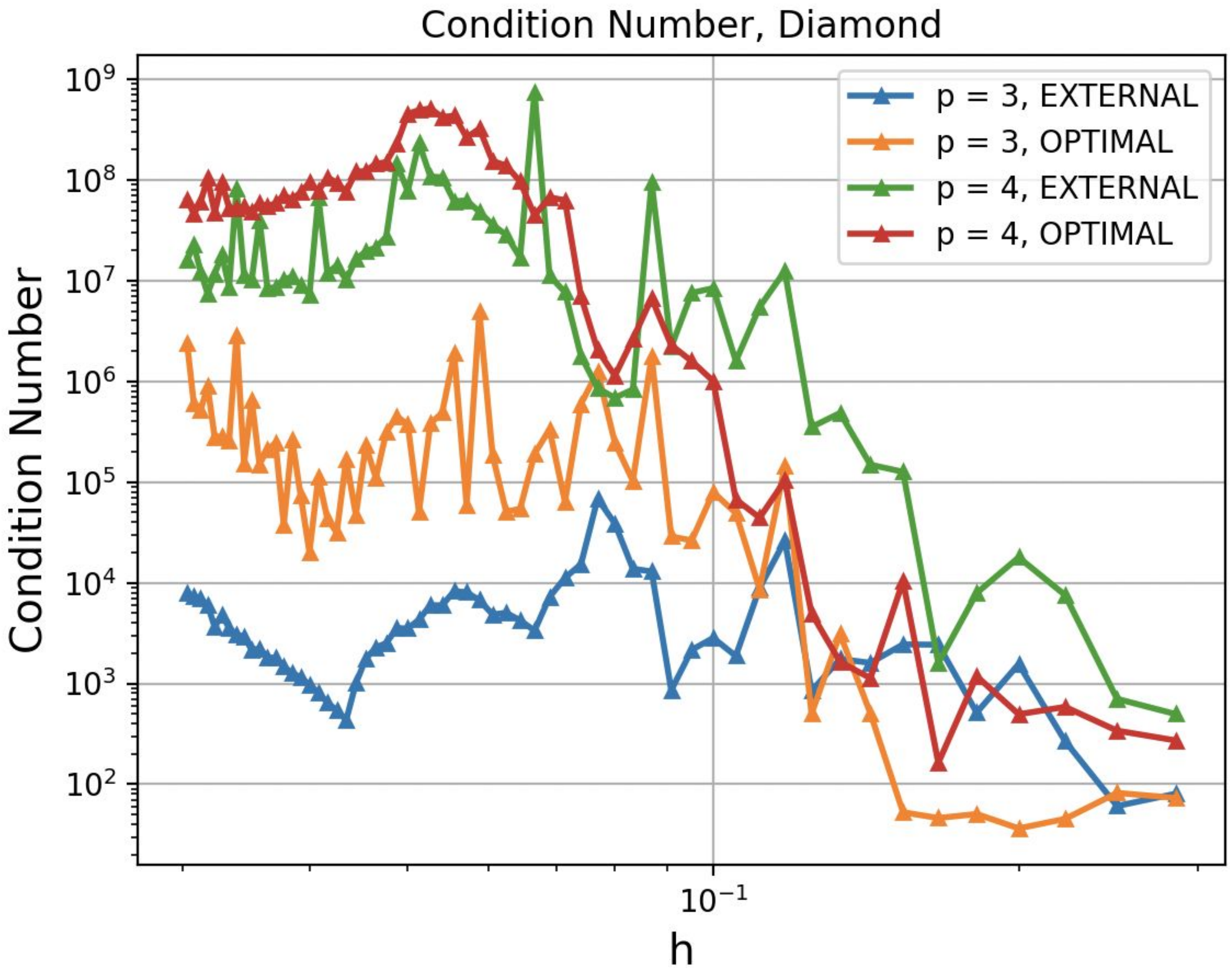
Results: ”Small active-support problem”

Why does the condition number explode when we do degree elevation?

There might be cases where a basis function has only a small portion of its support which is active [***Small active-support problem***].

For instance, when p = 4 the support of each basis function is 25 knot spans and might happen that only 1/25 is active and its small contribution causes instabilities.

(*Still work in progress …*)



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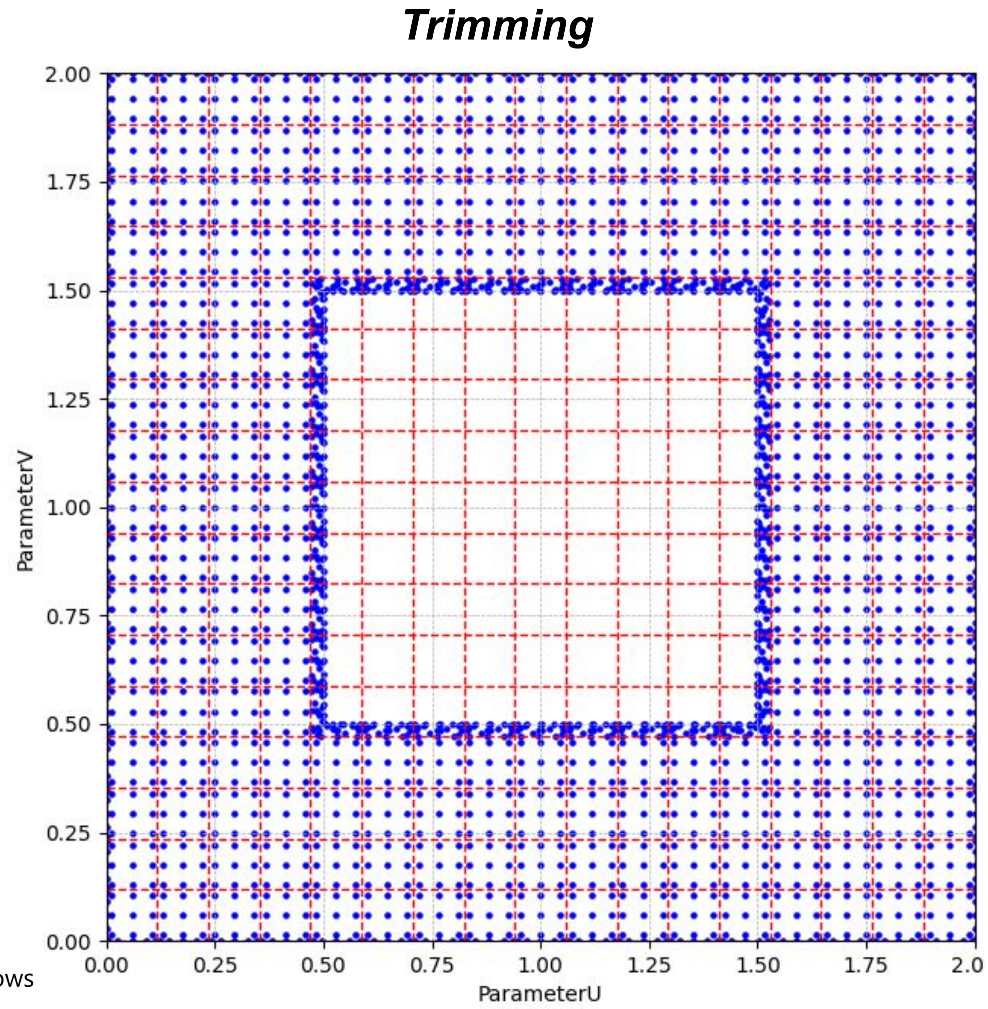
SBM vs Trimming

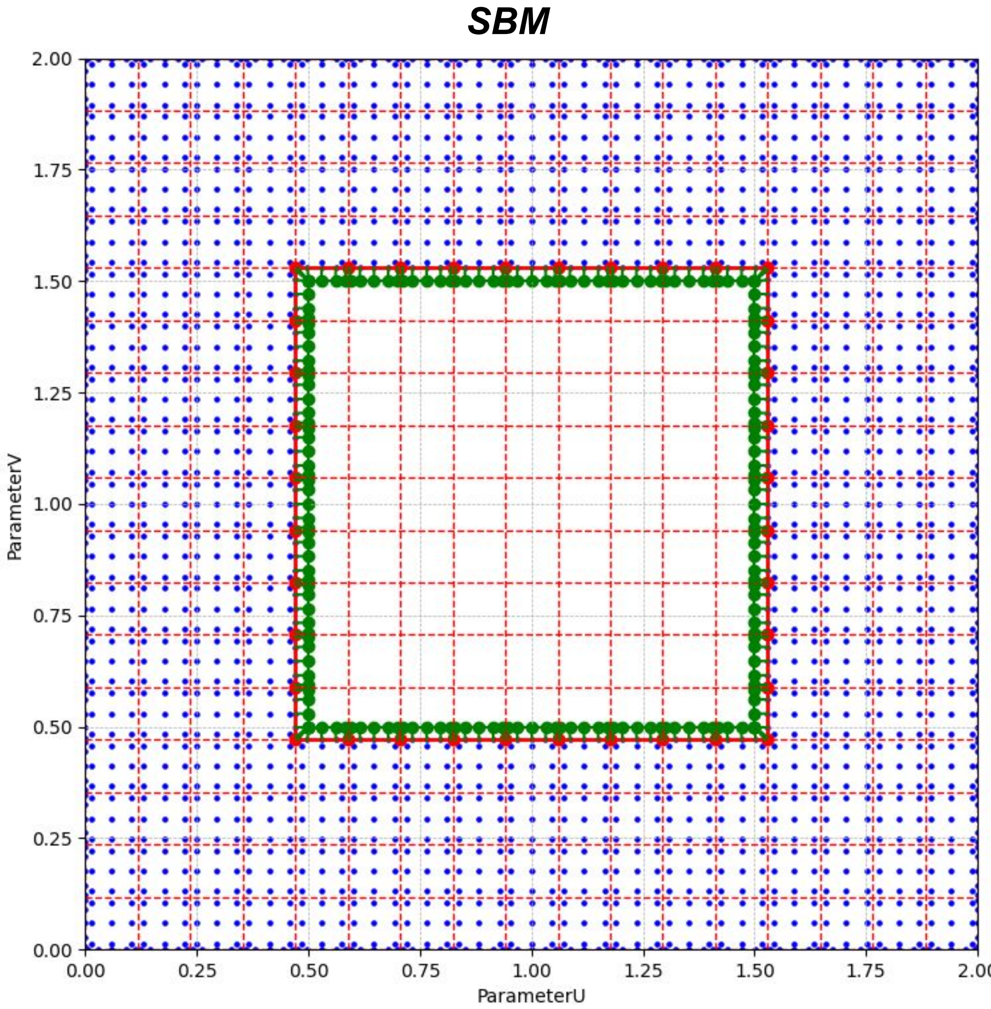
Trimming

Trimming is the technology that is now present in Kratos.

Using a ***tessellation technique*** we can integrate the “cut” knot spans.

(More details from Ricky Aristio in T1, January 2024)





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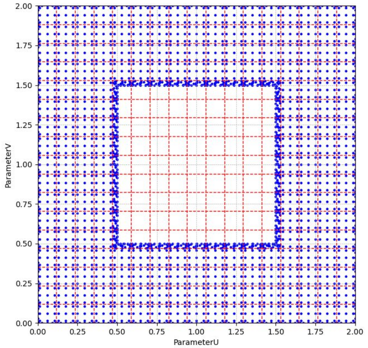
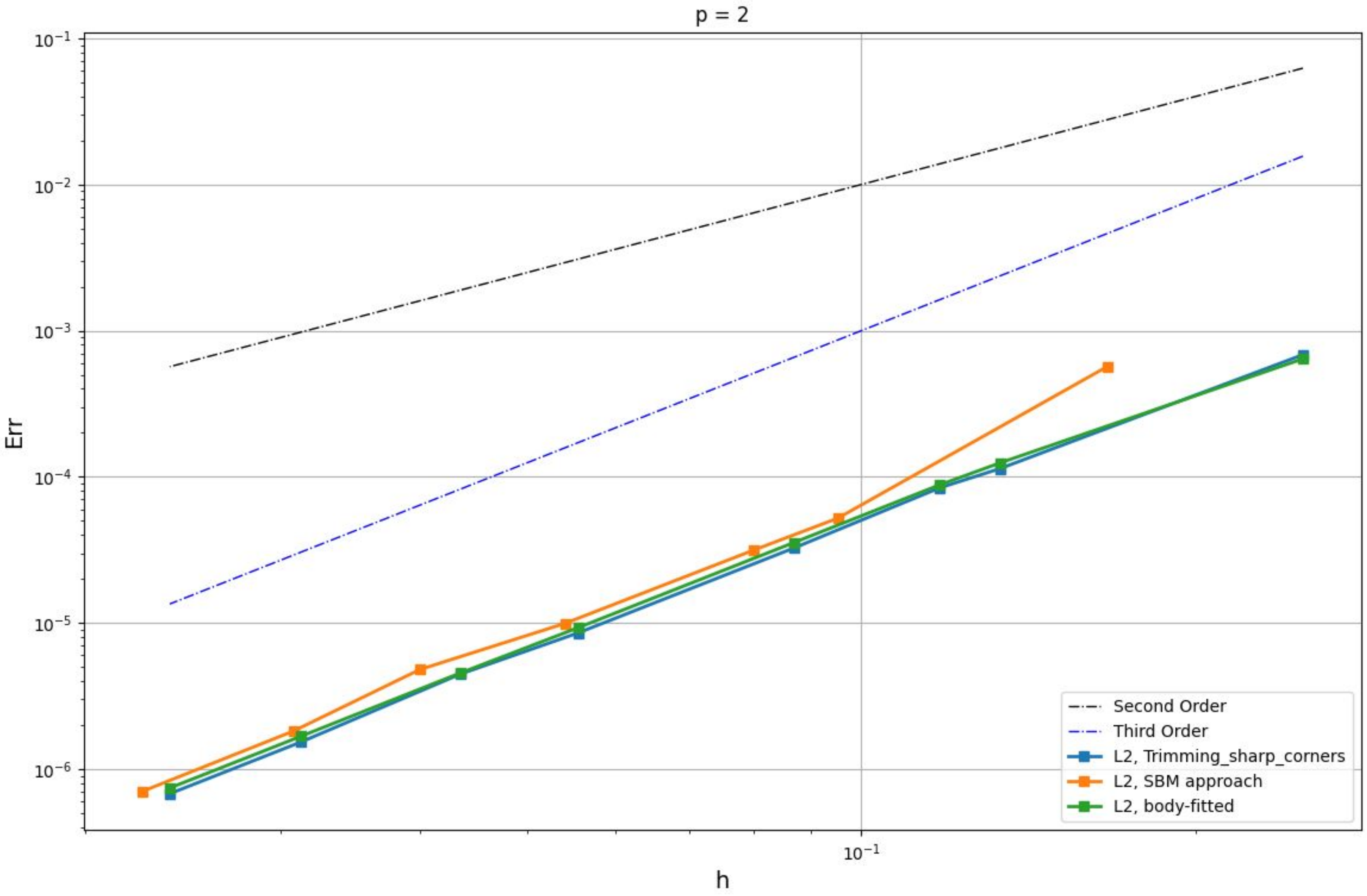
SBM vs Trimming

Comparison

Comparison between body-fitted, trimming and SBM approaches.

The polynomial order is p = 2 and we are using an embedded square.





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Future Work

- Optimize the implementation of the SBM technique in IGA.
- Pull request in the master of Kratos.
- Analysis of the “small active-support problem”
- Write a paper SBM in IGA for a Poisson problem

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Thank you!

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