CIMNE

EXCELENCIA SEVERO OCHOA

Gecko

Design for IGA-type discretization workflows

1st Technical Workshop

DC3 – Technical Advancements

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Date: 09/01/2024

Funded by the European Union

DC3: Contact Mechanics

Project title: *Application of IBRA-type discretizations in implicit contact mechanics*

Supervisors: Riccardo Rossi, Alejandro Cornejo Velazquez

Crash of a car against a deformable barrier, from Daimler Chrysler AG.

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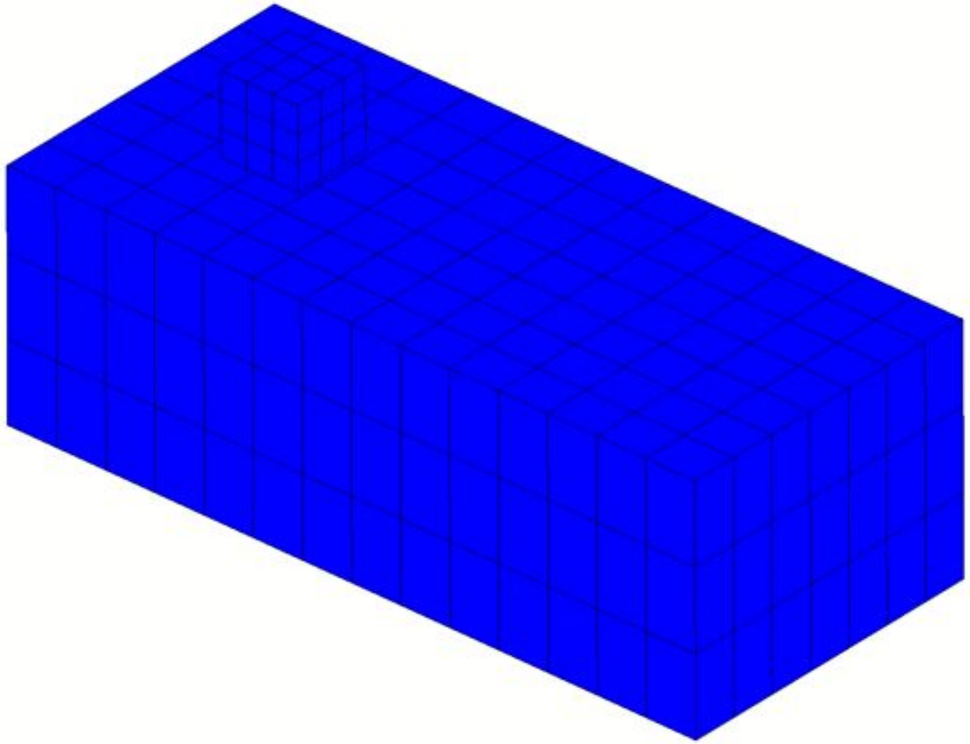
DC3: Contact Mechanics

Final Goal (3 years): *Solve 2 and 3 dimensional contact mechanics problems with IBRA-type discretizations and immersed methods (like the Shifted Boundary Method – SBM)*

Roadmap:

- ✔ *Study Isogeometric Analysis*
- ✔ *Solve simple 1D truss model in IGA + SBM*
- ☐ *Study the basis of Numerical Contact Mechanics*
- ✔ *Solve 1D contact model in IGA and IGA+SBM*
- ☐ *Understand available CM application in Kratos*
- ☐ *Study multipatch coupling*
- ☐ *Create a 2D contact model (plane stress) in Kratos*



Three-dimensional ironing problem with rotating indenter.
From Thomas Cichosz.

First steps in IGA: from simple truss to 1D contact

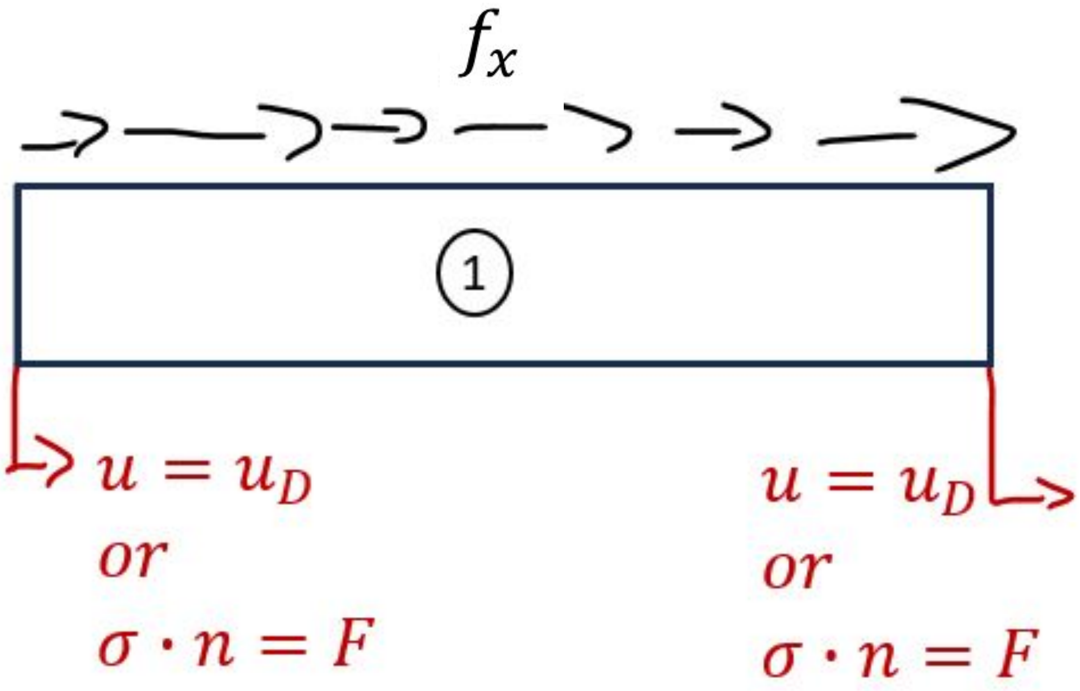
Governing equations

$$
\begin{aligned}
&div\,\boldsymbol{\sigma} + \hat{\boldsymbol{b}} = \rho\ddot{\boldsymbol{u}} && \text{in } \Omega \times [0,T],\\
&\boldsymbol{u} = \boldsymbol{u}_D && \text{on } \Gamma_D \times [0,T],\\
&\boldsymbol{\sigma} \cdot \boldsymbol{n} = \hat{\boldsymbol{t}} && \text{on } \Gamma_\sigma \times [0,T].
\end{aligned}
$$

$\boldsymbol{u}(x,y,z,t) = u_x$

Static
Linear Elastic

$$
\begin{aligned}
&\partial_x(E\partial_x u) + f_x = 0 && \text{in } [0,L],\\
&u = u_D && \text{on } \Gamma_D,\\
&E\,\partial_x u \cdot n = \mathrm{f_n} && \text{on } \Gamma_\sigma.
\end{aligned}
$$



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Testing: MANIFACTURED SOLUTIONS

Desired solution: $\mathbf{u}(x) = sin(x)/EA$ **given**

Distributed load (*internal forcing*)

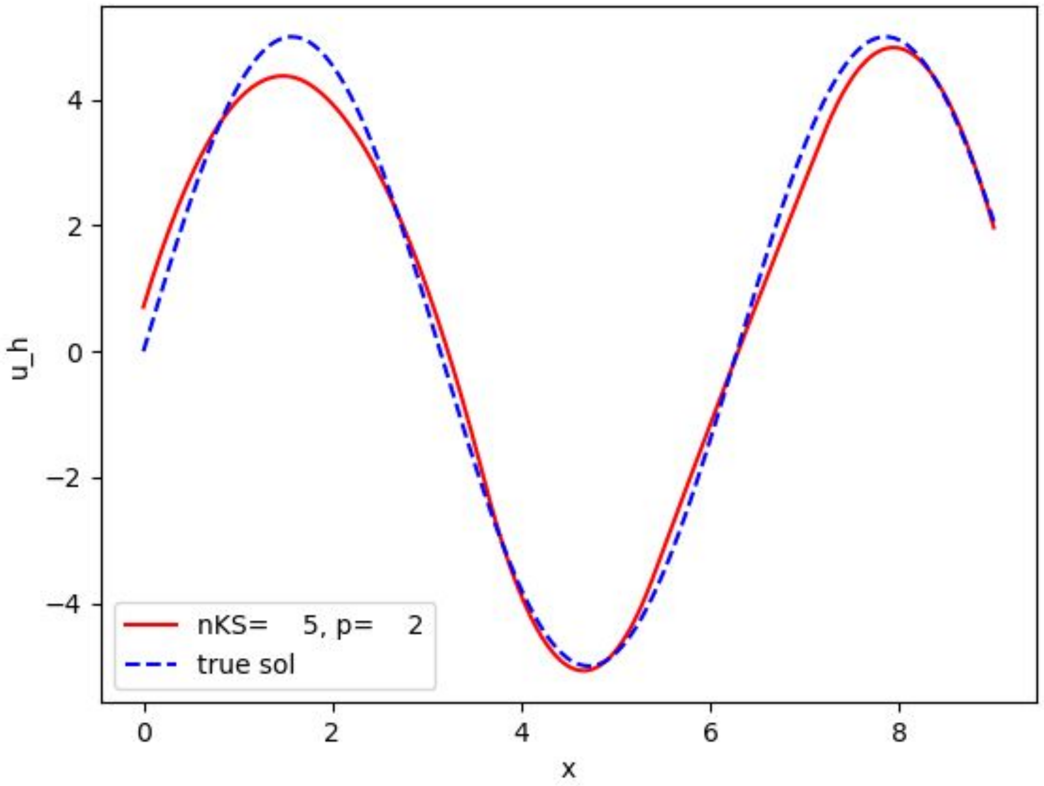
$$f = -\partial_x(E\,\partial u) \xrightarrow{E\,=\,const} f = -\frac{\sin(x)}{A} \xrightarrow{A\,=\,const} f_x = -\sin(x)$$

Concentrated load (*Neumann Condition*)

$$E\,\partial_x u \cdot n = f_n \Longrightarrow f_n = \frac{\cos(x_N)}{A} \xrightarrow{A\,=\,const} F = \cos(x_N)$$

Imposed Displacement (*Dirichlet Condition*)

$u = u(x_D)$ on Γ_D



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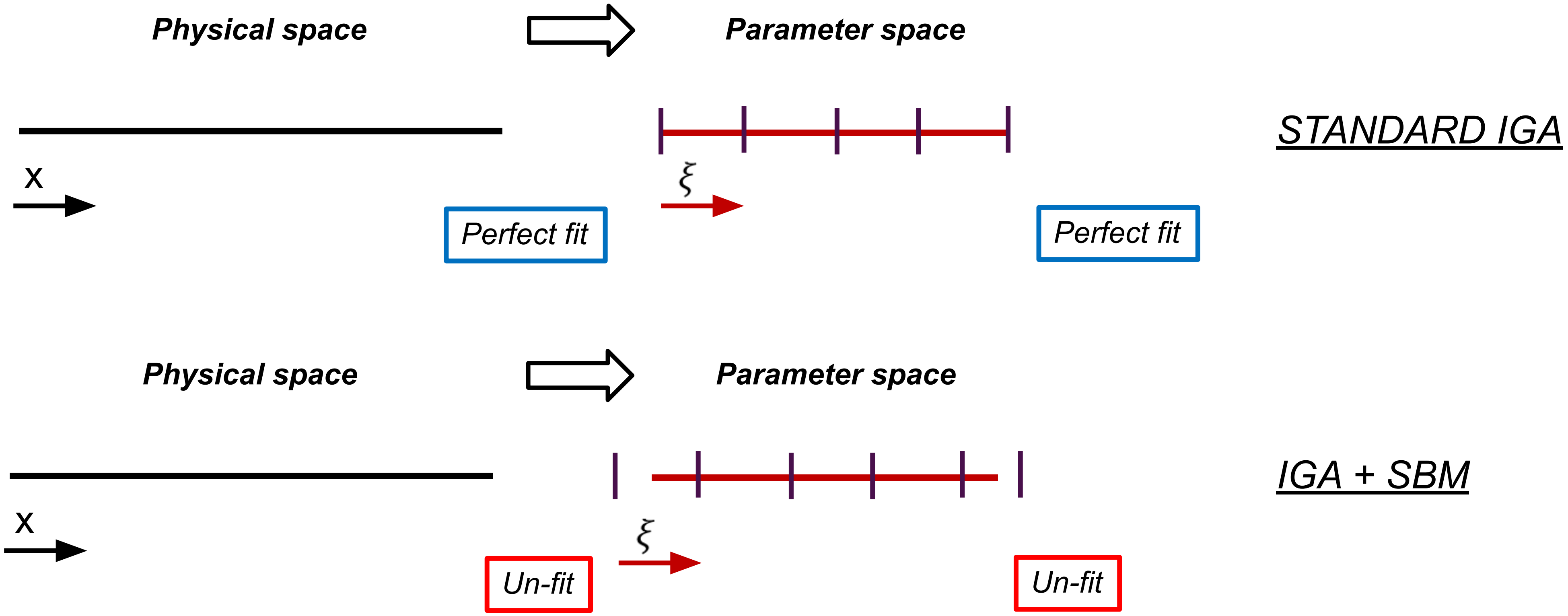
First steps in IGA: Shifted Boundary Method (SBM)



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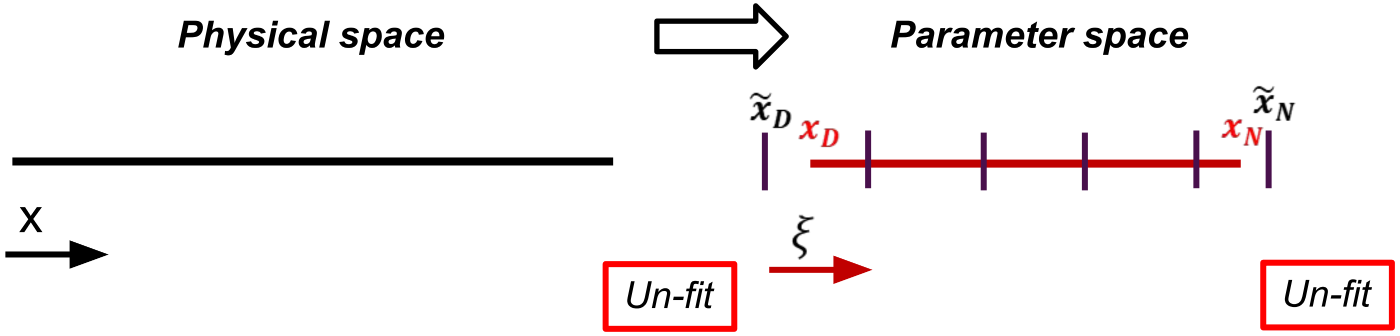
- SBM: *Modified condition at shifted boundary trough Taylor Expansion*

First steps in IGA: Shifted Boundary Method (SBM)



IGA + SBM

SBM DIRICHLET

$$u(x_D) = u(\tilde{x}_D) + \partial_x u(\tilde{x}_D) \cdot d + o(d^2)$$

$$\Rightarrow \quad u(\tilde{x}_D) = u(x_D) - \partial_x u(\tilde{x}_D) \cdot d + o(d^2).$$

$$d = x_D - \tilde{x}_D$$

SBM NEUMANN

$$\partial_x u(x_N) = \partial_x u(\tilde{x}_N) + \partial^2_{xx} u(\tilde{x}_N) \cdot d + o(d^2).$$

$$\Rightarrow \quad \partial_x u(\tilde{x}_N) = \partial_x u(x_N) - \partial^2_{xx} u(\tilde{x}_N) \cdot d + o(d^2).$$

$$d = x_N - \tilde{x}_N, \quad E\partial_x u(x_N) \cdot n = f_n$$

Neumann BC requires higher derivatives for the same approximation error!

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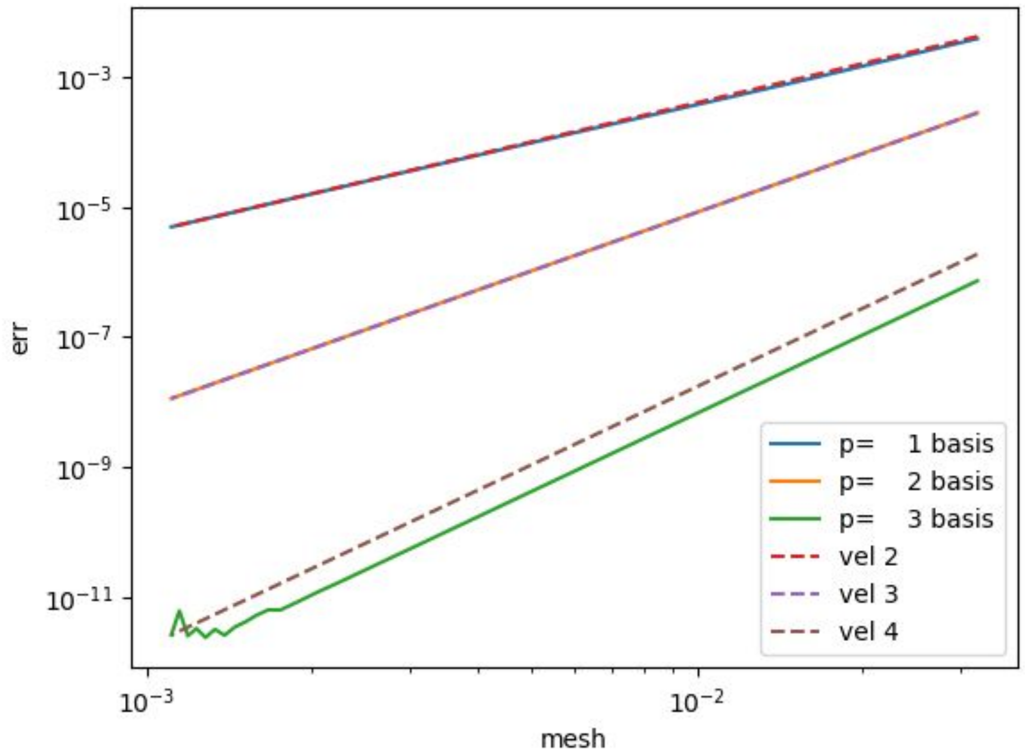
COMPARISON STANDARD IGA AND IGA+SBM

Standard IGA Convergence Error



Convergence rate $\sim h^{p+1}$

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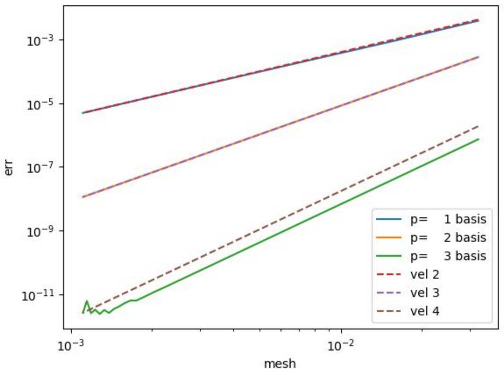
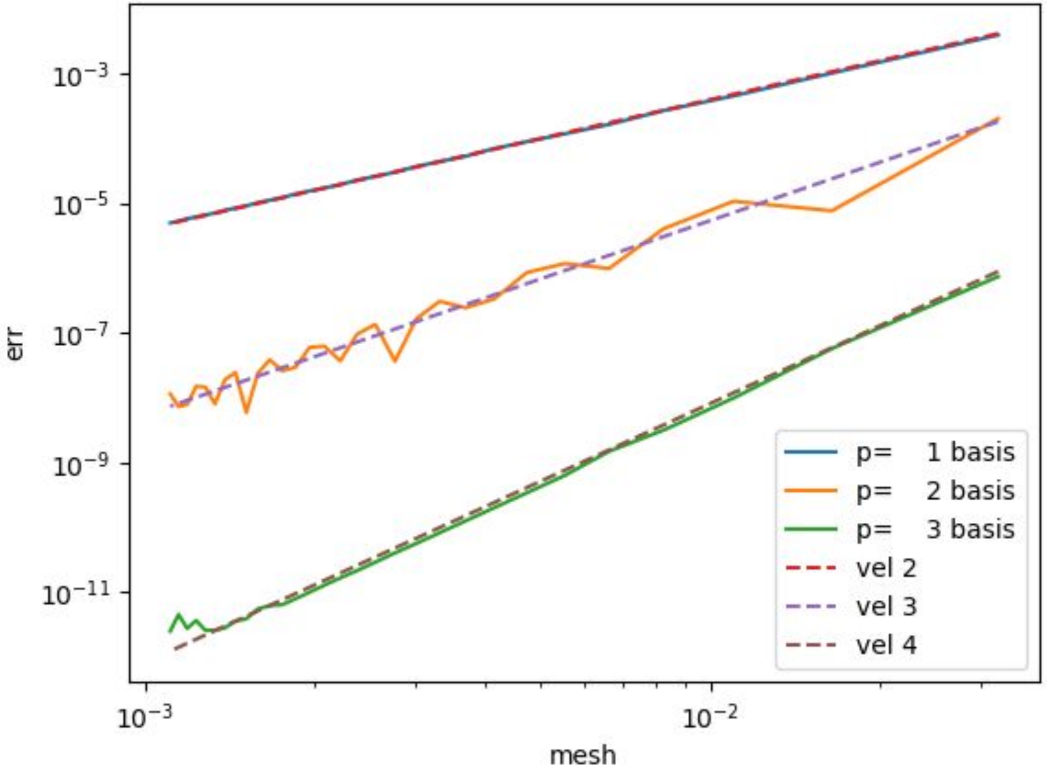
COMPARISON STANDARD IGA AND IGA+SBM

Standard IGA Convergence Error



IGA + SBM (Dirichlet)
Convergence Error



<u>Shifted Dirichlet Error</u>
(p derivatives available – Taylor expansion of order $p+1$)

Convergence rate $\sim h^{p+1}$

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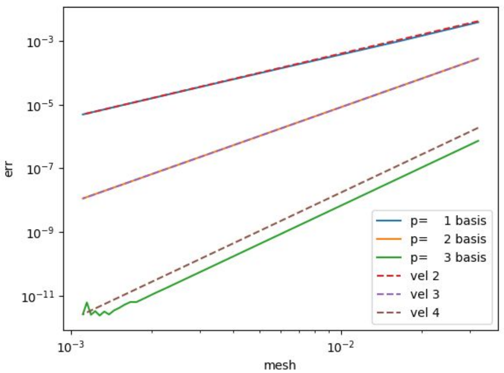
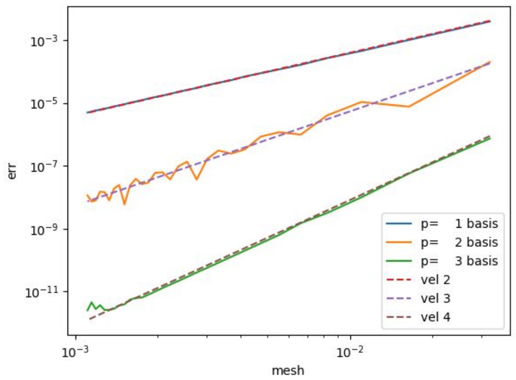
COMPARISON STANDARD IGA AND IGA+SBM

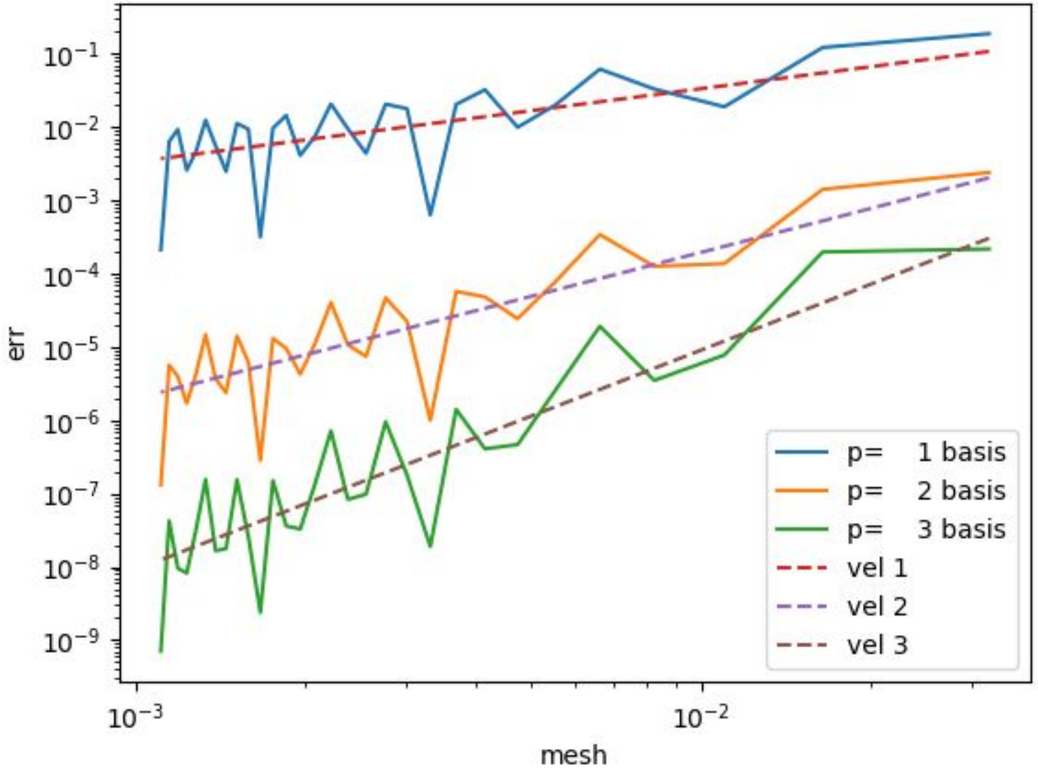
Standard IGA Convergence Error



IGA + SBM (Dirichlet) Convergence Error



IGA + SBM (Complete) Convergence Error



<u>Shifted Dirichlet Error</u>

(p derivatives available – Taylor expansion of order p+1)

<u>Shifted Neumann Error</u>

(p derivatives available – Taylor expansion of order p)

Convergence rate $\sim h^p$

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Contact Mechanics: Brief Overview

Project title: *Application of IBRA-type discretizations in implicit contact mechanics*

$$div\,\boldsymbol{\sigma} + \hat{\boldsymbol{b}} = \rho\ddot{\boldsymbol{u}} \quad \text{in } \Omega \times [0,T],$$
$$\boldsymbol{u} = \boldsymbol{u}_D \quad \text{on } \Gamma_D \times [0,T],$$
$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \hat{\boldsymbol{t}} \quad \text{on } \Gamma_\sigma \times [0,T].$$

Governing equations (object 1,2,…,N)

$$g_n(\boldsymbol{X},t) \geq 0, \qquad p_n(\boldsymbol{X},t) \leq 0,$$
$$p_n(\boldsymbol{X},t)g_n(\boldsymbol{X},t) = 0.$$

Normal Contact Constraints

$$\boldsymbol{\Phi} = \left|\left|\boldsymbol{t}_\tau\right|\right| - \mu|p_n| \leq 0\,, \; \boldsymbol{v}_{\tau,rel} + \beta\boldsymbol{t}_\tau = \boldsymbol{0},$$
$$\beta \geq 0, \qquad \boldsymbol{\Phi}\beta = 0$$

Frictional Contact Constraints (Coulomb)

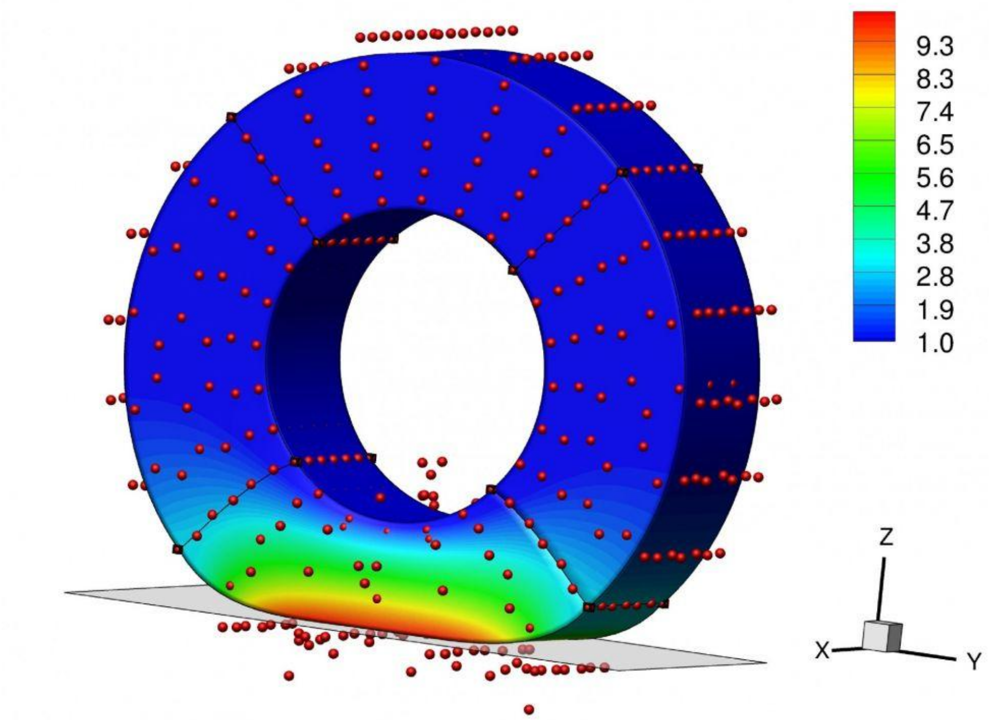

Image source:
https://me.bilkent.edu.tr/7-research/21-projects/348-isogeometric-computational-contact-mechanics-rta3/

Extra:
Material/geometric nonlinearities
Thermal problem
Adhesion/lubrification,… (additional interface conditions)

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Contact Mechanics: Brief Overview

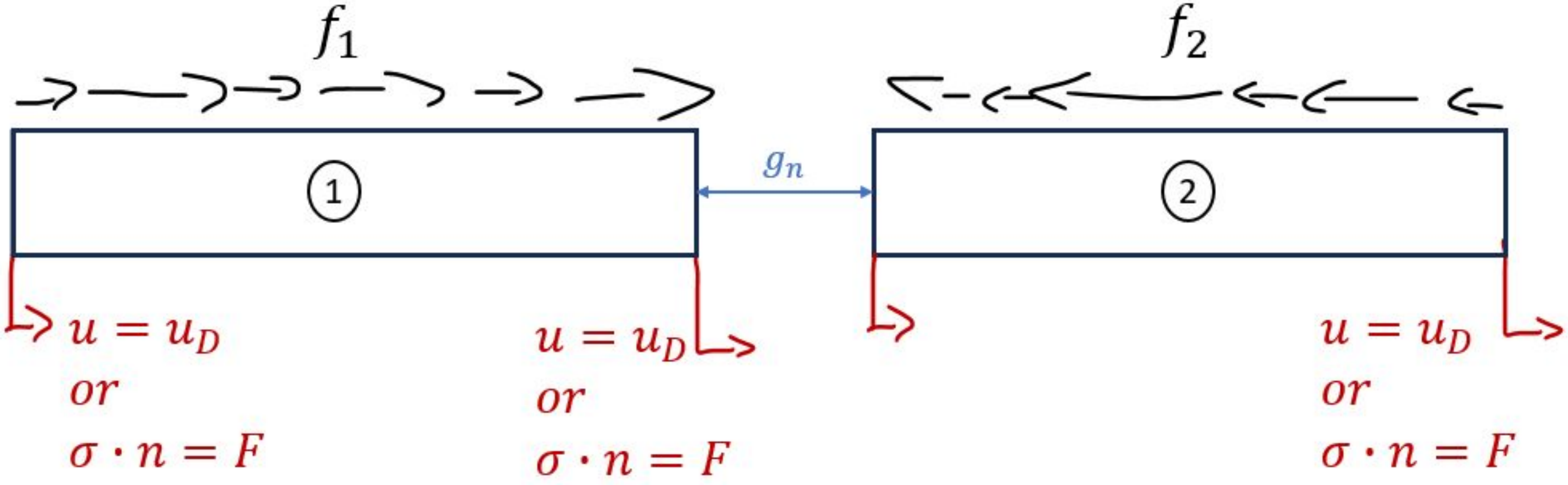
Problem: *1D contact between trusses*

$$
\begin{aligned}
&div\, \boldsymbol{\sigma} + \hat{\boldsymbol{b}} = \rho \ddot{\boldsymbol{u}} && \text{in } \Omega \times [0,T], \\
&\boldsymbol{u} = \boldsymbol{u_D} && \text{on } \Gamma_D \times [0,T], \\
&\boldsymbol{\sigma} \cdot \boldsymbol{n} = \hat{\boldsymbol{t}} && \text{on } \Gamma_\sigma \times [0,T].
\end{aligned}
$$

Governing equations
(object 1,2,…,N)

$$
g_n(\boldsymbol{X},t) \geq 0, \qquad p_n(\boldsymbol{X},t) \leq 0,
$$
$$
p_n(\boldsymbol{X},t) g_n(\boldsymbol{X},t) = 0.
$$

Normal Contact Constraints



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Contact Mechanics: Brief Overview

Constraint enforcement: *Lagrange and Penalty methods for the frictionless model*

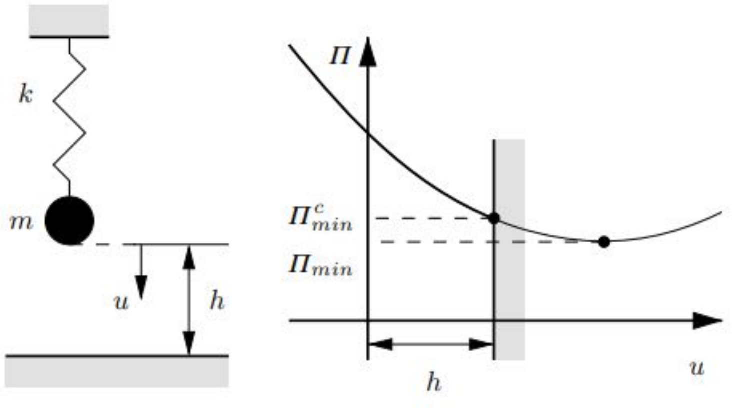
Hyp. Linear Elasticity Governing Equations: $\partial\Pi = 0$

Contact Governing Equations: $\partial\Pi^c = 0$

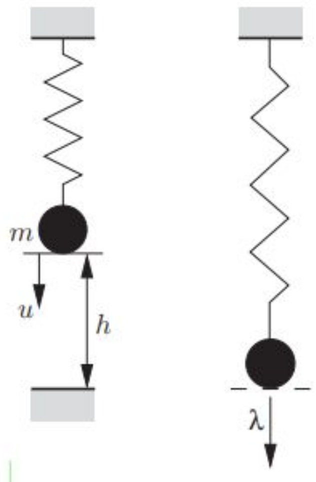
$$\Pi^c = \Pi + \Pi^*$$

$$\Pi^* = \int_{\Gamma_C} \lambda g_n$$

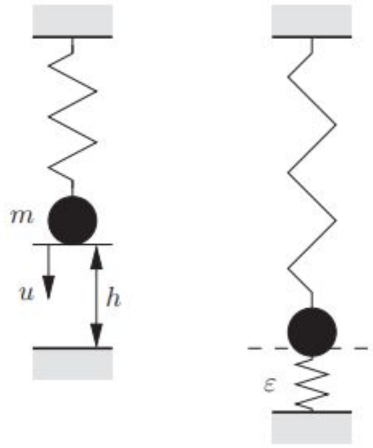
$$\Pi^* = \frac{1}{2}\int_{\Gamma_C} \epsilon g_n^2$$



General formulation



Lagrange formulation



Penalty formulation

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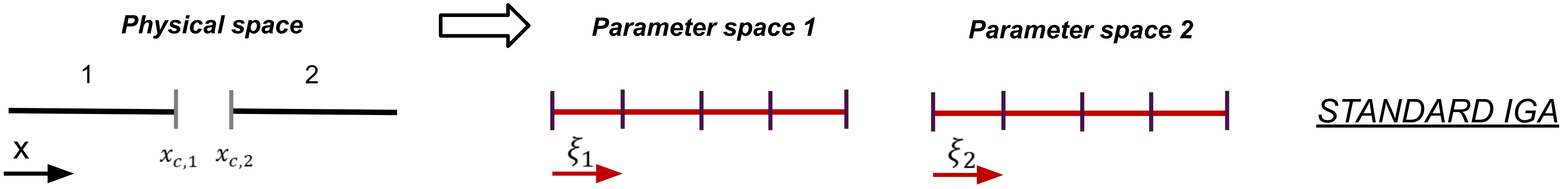
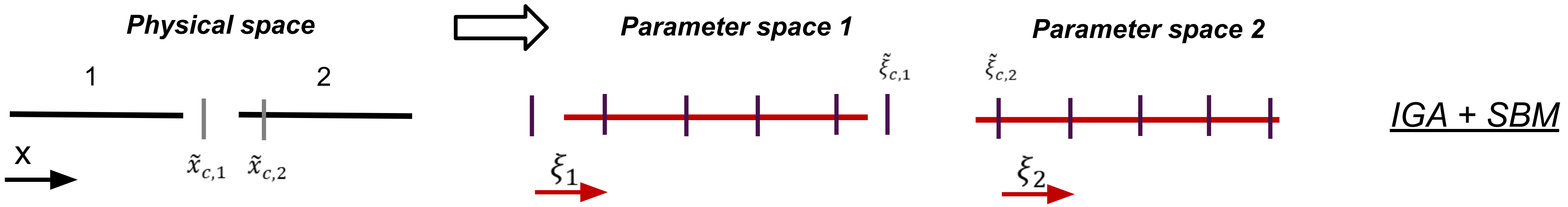
IGA + SBM: Contact Discretization





- SBM: *Modified condition at shifted boundary trough Taylor Expansion*

$$g_n(x) = g_{n,0} + (u_2(x_{c,2}) - u_1(x_{c,1})) \cdot n$$

$$u_1(x_{c,1}) = u_1(\tilde{x}_{c,1}) + \nabla u_1(\tilde{x}_{c,1}) \cdot (x_{c,1} - \tilde{x}_{c,1}) + \ \dots$$

$$u_2(x_{c,2}) = u_2(\tilde{x}_{c,2}) + \nabla u_2(\tilde{x}_{c,2}) \cdot (x_{c,2} - \tilde{x}_{c,2}) + \ \dots$$

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Testing: MANIFACTURED SOLUTIONS

Desired solution: $u_1(x)$, $u_2(x), F_C$ given

Distributed load (*internal forcing*)

$$f_i = -\partial_x(E_i\,\partial u_i) \xrightarrow{E_i = const} f_i = -E_i \partial^2_{xx}\, u_i \xrightarrow{A_i = const} f_{i,x} = -A_i E_i \partial^2_{xx}\, u_i$$

Concentrated load (*Neumann Condition if not contact boundary*)

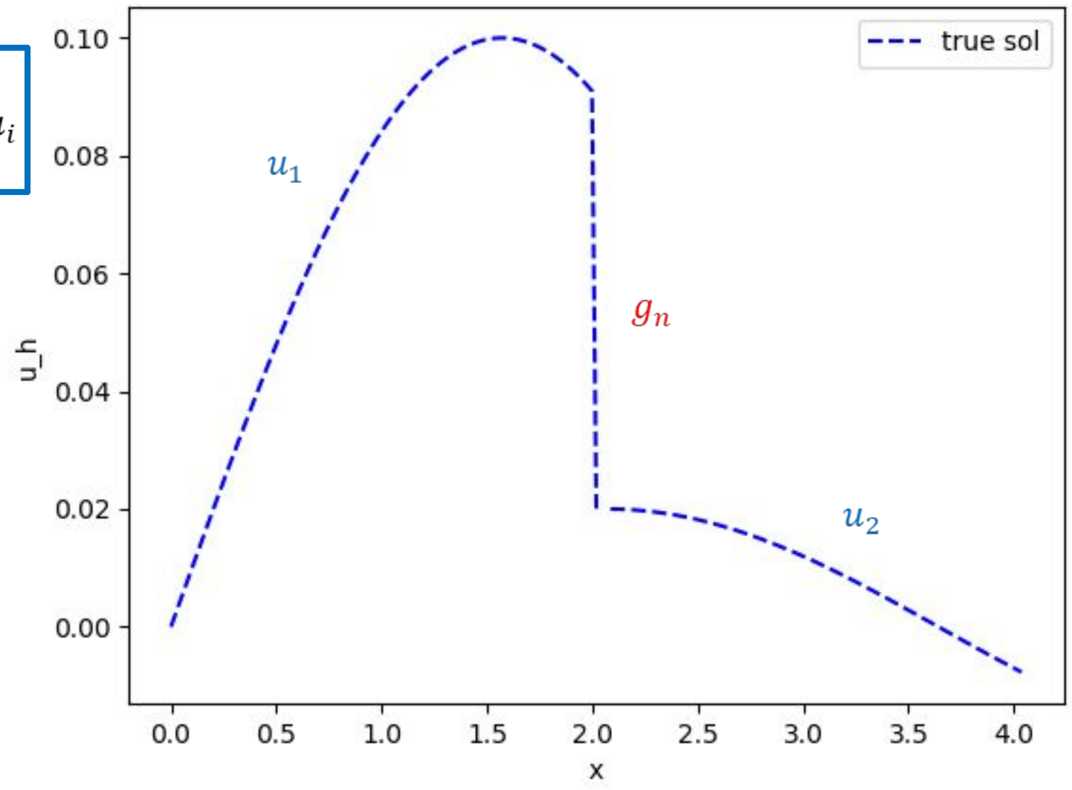
$$E_i\,\partial_x u_i \cdot n = f_{n,i} \xrightarrow{A = const} F_i = A_i E_i\,\partial_x u_i \cdot n$$

Imposed Displacement (*Dirichlet Condition*)

$u_i = u_i(x_D)$ on $\Gamma_{i,D}$

Contact condition

$$f_{n,1} A_1 = F_1 - F_C \quad \& \quad f_{n,2} A_2 = F_2 + F_C$$
$$g_n = u_1(L_1) - u_2(0)$$



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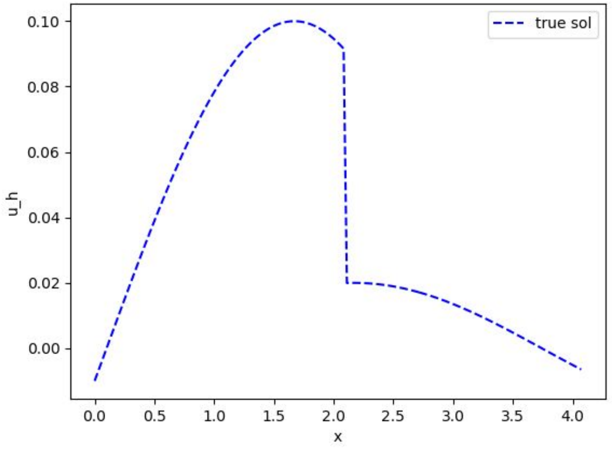
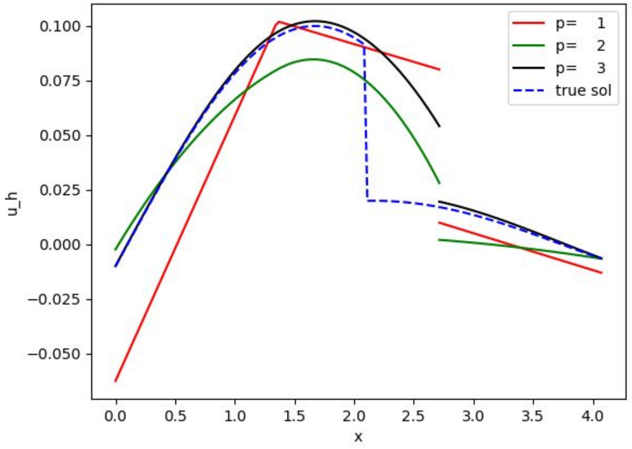
COMPARISON STANDARD IGA AND IGA+SBM

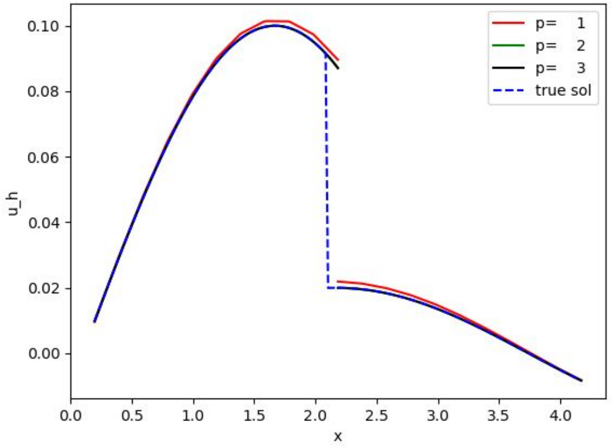
STANDARD IGA-CONVERGENCE



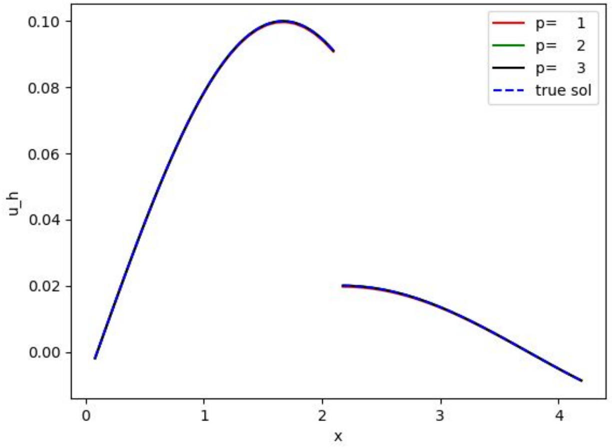
True sol



3 knot spans



20 knot spans



50 knot spans

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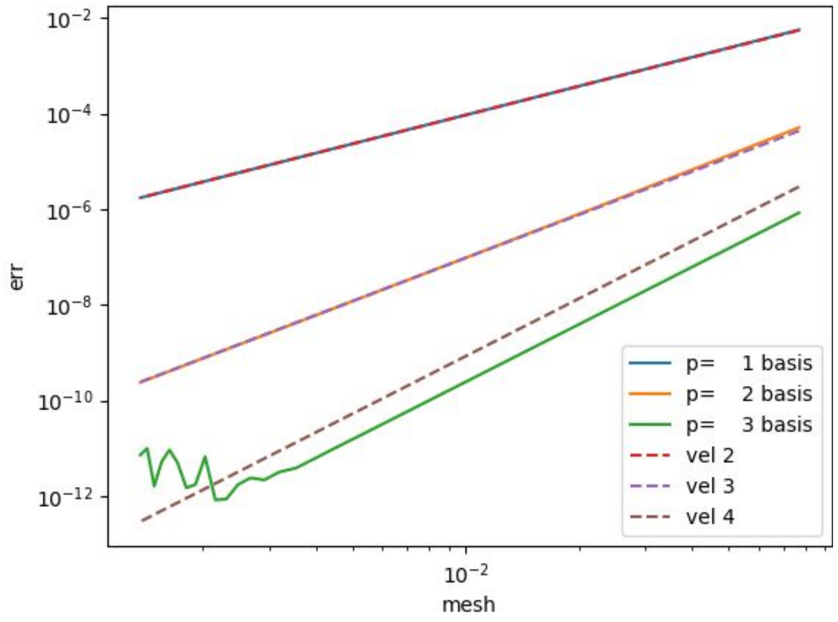
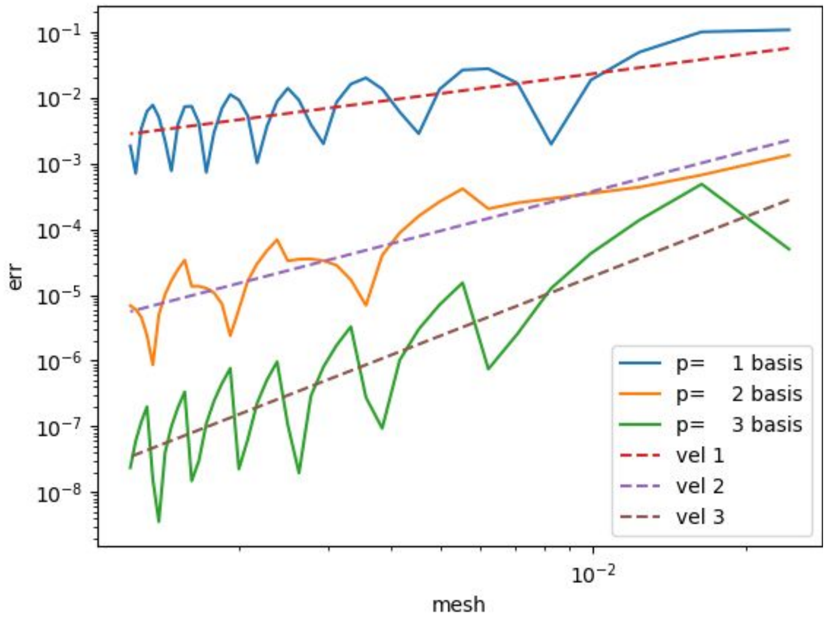
COMPARISON STANDARD IGA AND IGA+SBM

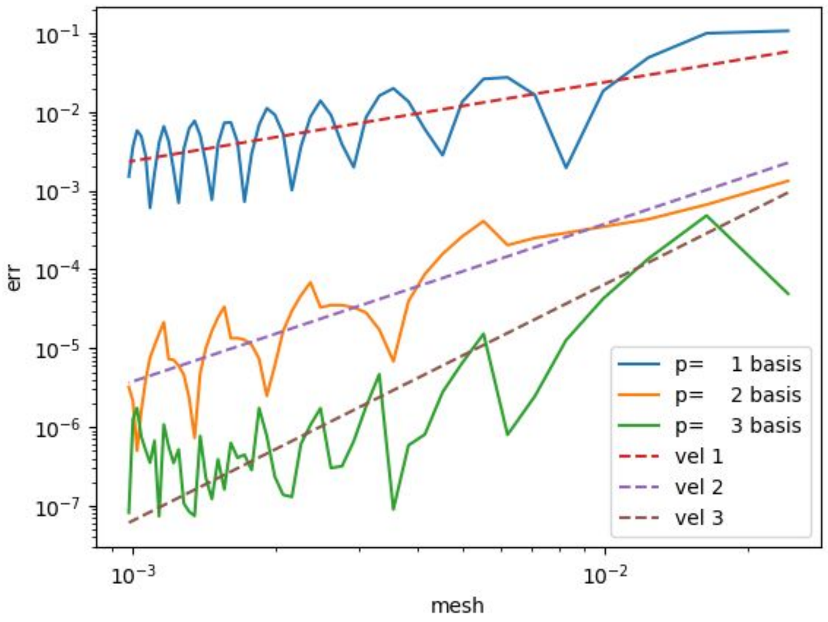
Standard IGA



IGA + SBM (Lagrange)



IGA + SBM (Penalty)



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DC3 – Close term development plan

Application of IBRA-type discretizations in implicit contact mechanics

Supervisors: Riccardo Rossi, Alejandro Cornejo Velazquez

Upcoming steps

- Understand available CM application in Kratos
- Develop an IGA contact mechanics application in 2D
- Extend the analysis with the Shifted Boundary Method

KRATOS MULTI-PHYSICS

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European Commission

Thank you!

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