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Design for IGA-type
discretization workflows

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1st Technical Workshop

DC09 – Research progress meeting

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Recap: DC09 - Research field

Model Order Reduction of coupled vibro-acoustic systems

Objectives:

- Combination of IGA and FEM/BEM for vibro-acoustic systems
- Explore Model-Order Reduction (MoR) schemes to IGA-iBEM for coupled vibro-acoustic methods
- Incorporate Fast-Multipole and H-matrix approaches within IGA-BEM MoR framework

➡ **Considering one domain- acoustic domain- for now □ IGA-BEM**

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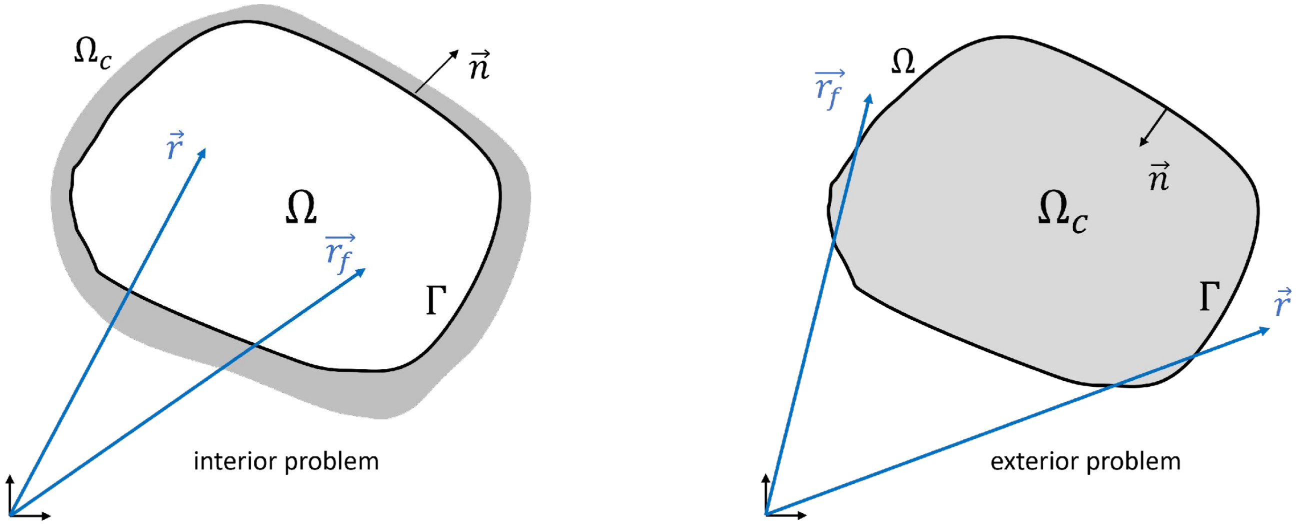
Fundamentals of Computational Acoustics

Acoustic Basics (I)

- For acoustic problems: $\Delta p(\vec{r}) + k^2 p(\vec{r}) = -j\rho_0 \omega q_a \delta(r, r_q)$

Helmholtz equation (Hermann Ludwig Ferdinand von Helmholtz 1821-1894)



interior problem

exterior problem

Ω	computational domain
Γ	boundary of Ω and Ω_c
$\vec{r}, \vec{r_f}$	position vectors
Ω_c	complementary domain
$\vec{n}$	normal vector

Figure 1.: Description of a general acoustic problem

Fundamentals of Computational Acoustics

Acoustic Basics + IGA

- In literature: direct and indirect methods
- Variational formulation of the sound pressure $p(\vec{r})$:

$\mu(r_f)$: *double layer potential*

$\sigma(r_f)$: *single layer* potential

$$p(\vec{r}) = \int_{\Omega_f} \left(\mu(r_f) \frac{\partial G(r, r_f)}{\partial n} - \sigma(r_f) G(r, r_f) \right) d\Omega_f(r_f)$$

- Advantage of indirect BEM (iBEM): Combined interior / exterior problems, e.g. open boundaries can be solved
- Approximation of single- and double-layer potentials $\sigma(r_f)$ and $\mu(r_f)$:

$$\sigma_{r_f} \approx \sigma^h(r_f) = \sum_{i=1}^{n} N_i(r_f) d_i^{\sigma} \qquad r_f \in \Gamma_\sigma$$

$$\mu_{r_f} \approx \mu^h(r_f) = \sum_{i=1}^{n} N_i(r_f) d_i^{\mu} \qquad r_f \in \Gamma_\mu$$

where:		
	n	number of NURBS basis functions
	$N_i(r_f)$	basis function
	d_i	control point

Motivation of Research

Model Order Reduction of coupled vibro-acoustic systems

Why MoR IGA-BEM?

- Only boundary needs to be discretized in BEM compared to FEM
- Fulfillment of Sommerfeld radiation condition at infinity for BEM
- Sensitive to geometric description of surface Incorporation of IGA with BEM
- However: There is no free lunch! (theorem of conservation of difficulties)
- System matrices in BEM are **dense, non-affine and highly oscillatory!**
- Use MoR IGA-BEM to reduce computational cost

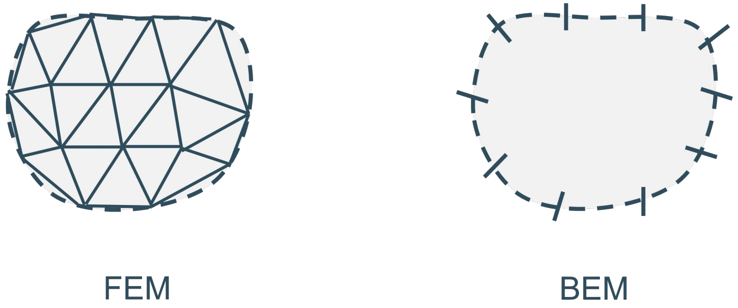


Figure 2: Overview of numerical methods

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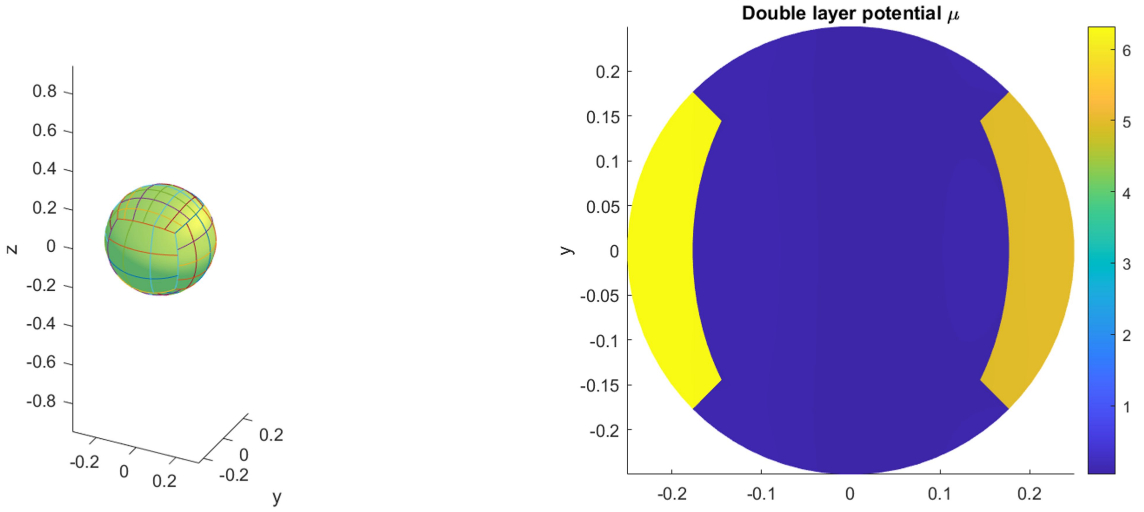
IGA-iBEM Implementation

Computation of the double layer potential (I)

- Patches in general have non-conforming discretizations
- Patch coupling needed!
- Herein: Patch coupling by virtual refinement in a master-slave formulation [Co16]
- Virtual refinement of interface DOFs until patches are conforming
- Use substitution method for condensation of slave DoFs



(a) (b)

Figure 3.: (a) 3D sphere, (b) Top view double layer potential μ at 100 Hz without enforcement

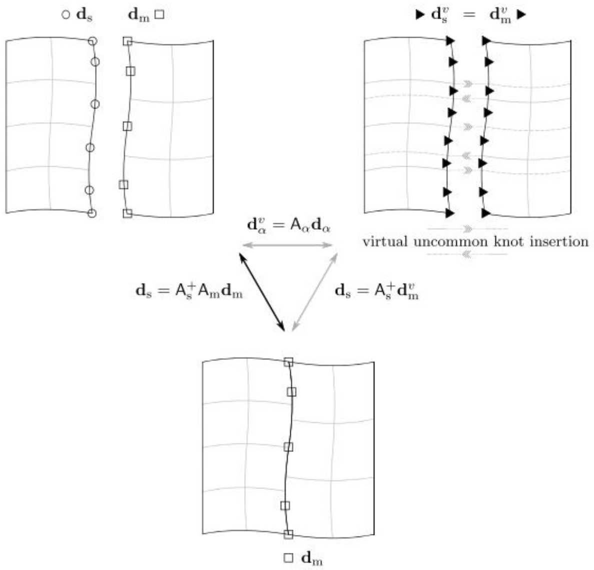


Figure 4.: Conceptual illustration of patch coupling method [Co16]

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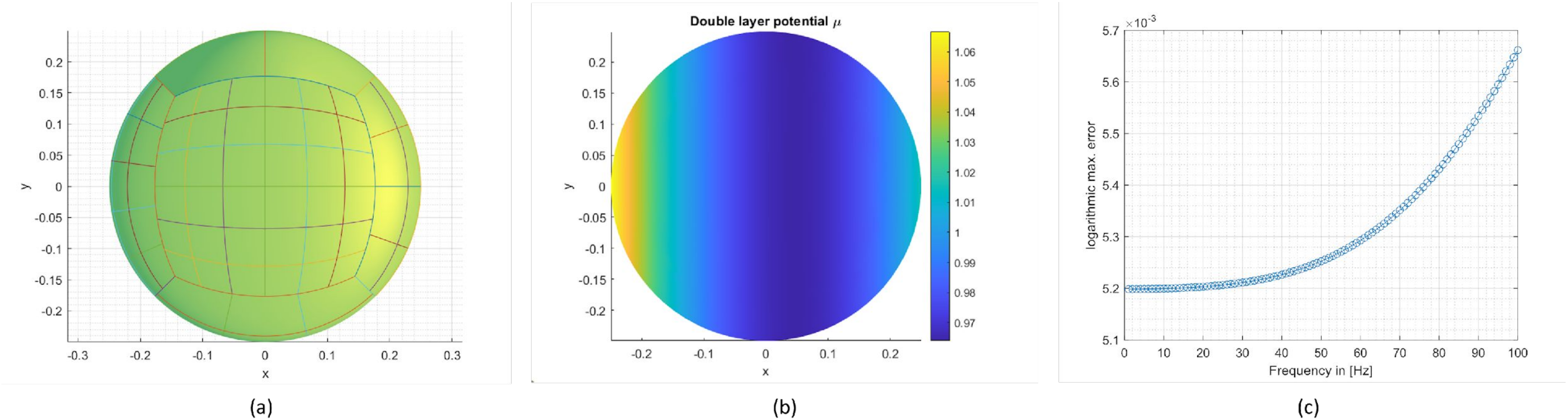
IGA-iBEM Implementation

Computation of the double layer potential (II)



(a) (b) (c)

Figure 5.: (a) non-conforming multipatch geometry: sphere; (b) double layer μ potential at 100 Hz; (c) logarithmic maximum error of the sound pressure over frequency

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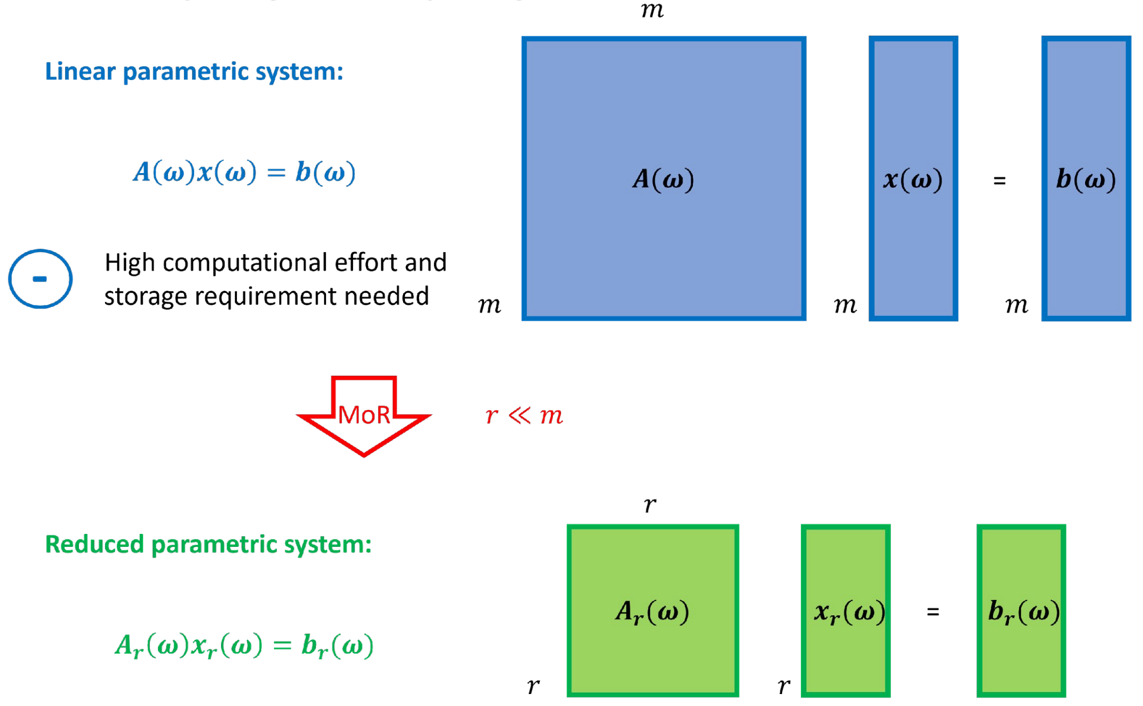
Model-Order Reduction of BEM systems

Problem of BEM systems

- BEM system expressed as: $A(\omega)x(\omega) = b(\omega)$ $\quad$ $where\ A: \Psi \rightarrow \mathbb{C}^{NxN}$ and $\mathrm{x}, \mathrm{b}: \Psi \rightarrow \mathbb{C}^{N}$
- Dealing with **dense**, **non-affine** and **highly oscillatory** matrices
- Increase of computational time and memory storage with increase of complexity and frequency!

➡ Model Order Reduction (MoR)

- Find lower-order model that approximates the original high-order model, where the lower-order model facilitates both computationally efficiency and accuracy



Linear parametric system:

$\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}(\boldsymbol{\omega}) = \boldsymbol{b}(\boldsymbol{\omega})$

- High computational effort and storage requirement needed

Reduced parametric system:

$\boldsymbol{A_r}(\boldsymbol{\omega})\boldsymbol{x_r}(\boldsymbol{\omega}) = \boldsymbol{b_r}(\boldsymbol{\omega})$

Figure 6.: Schematic scheme projection MoR

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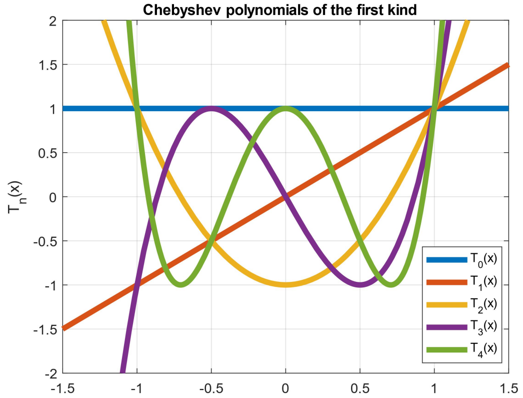
Model-Order Reduction within IGA-framework

Idea

- Combination of standard MoR schemes with BEM are not directly applicable due to the non-affine characteristic
- Cumbersome to construct representative basis + after obtaining representative basis a reduction of computational effort is not guaranteed for a frequency sweep analysis
- Responsible for non-affine characteristic: Green's function $G = \frac{1}{4\pi}\frac{e^{ikr}}{r} \quad where\ k = \frac{\omega}{c}$

Idea [Pa20]:

- Approximation of BEM system by an affine expression:
- Herein: Chebyshev polynomial Approximation



$$A(\omega) \approx \left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) T_i\right); \quad b(\omega) \approx \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) q_i \quad \rightarrow \left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) T_i\right) x(\omega) = \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) q_i$$

$$T_i = \frac{2}{\mathcal{M}+1} \sum_{\kappa=0}^{\mathcal{M}} A(\omega_\kappa) c_i(\omega_\kappa) \qquad q_i = \frac{2}{\mathcal{M}+1} \sum_{\kappa=0}^{\mathcal{M}} b(\omega_\kappa) c_i(\omega_\kappa) \qquad \omega_\kappa = \cos\left[\frac{\pi\left(\kappa+\frac{1}{2}\right)}{\mathcal{M}+1}\right]$$

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Model-Order Reduction within IGA-framework

Recycling of Krylov subspaces (I)

- Apply Galerkin projection:

$$x(\omega) \approx \hat{x}(\omega) = V\tilde{x}(\omega) \qquad b(\omega) \quad where\ V \in \mathbb{C}^{m\times r} \qquad r \ll m$$

- Obtain projection basis V for all $\omega \in \Psi$ by recycling Krylov subspaces
- Subspace recycling refers that the Krylov subspaces of the i^{th} system are utilized for accelerating the convergence of iterative solution procedure of the $(i+1)^{st}$ system
- Expand Krylov subspaces $\mathcal{K}_m^{\omega_i}$ of dimension m for all ω and generate basis V_{ω_i}
- Projection basis V constructed by SVD factorization

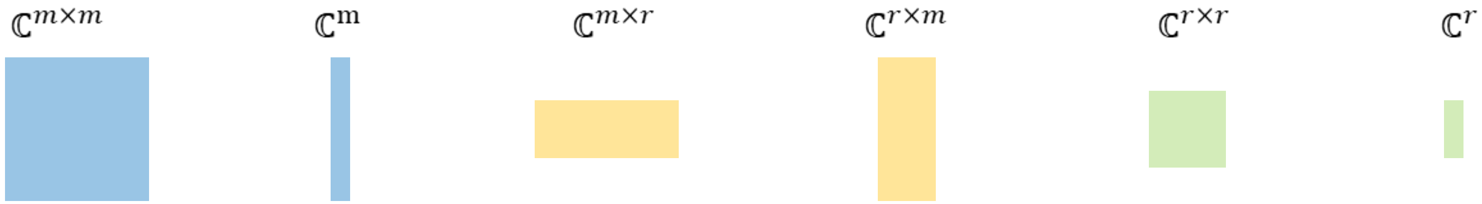
$$A_{red}(\omega) \approx \left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) T_{i,red}\right); \; b_{red}(\omega) \approx \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) q_{i,red} \;\rightarrow \left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) T_{i,red}\right) x(\omega) = \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) q_{i,red}$$

$$T_i = \frac{2}{\mathcal{M}+1} \sum_{\kappa=0}^{\mathcal{M}} W A(\omega_\kappa) V c_i(\omega_\kappa) \qquad q_i = \frac{2}{\mathcal{M}+1} \sum_{\kappa=0}^{\mathcal{M}} W b(\omega_\kappa) c_i(\omega_\kappa) \qquad \omega_\kappa = \cos\left[\frac{\pi\left(\kappa+\frac{1}{2}\right)}{\mathcal{M}+1}\right]$$

Model-Order Reduction within IGA-framework

Recycling of Krylov subspaces (I)

- Introducing:



- Full order model Chebyshev polynomial approximation:

$$\left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega)\, \boldsymbol{T_i}\right) x(\omega) = \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega)\, \boldsymbol{q_i}$$

$$T_i = \frac{2}{\mathcal{M}+1}\sum_{\kappa=0}^{\mathcal{M}} \boldsymbol{A(\omega_\kappa)}\; c_i(\omega_\kappa) \qquad q_i = \frac{2}{\mathcal{M}+1}\sum_{\kappa=0}^{\mathcal{M}} \boldsymbol{b}(\boldsymbol{\omega_\kappa})\; c_i(\omega_\kappa)$$

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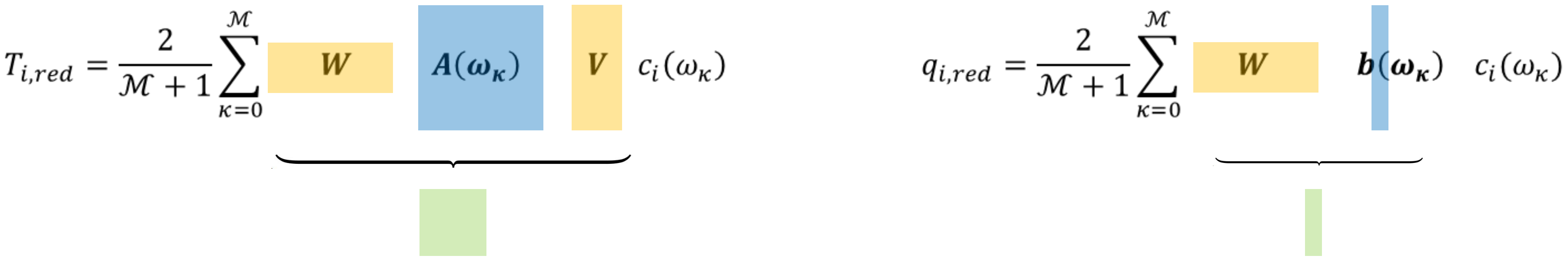
Recycling of Krylov subspaces (I)

- Reduced order model Chebyshev polynomial approximation:



$$T_{i,red} = \frac{2}{\mathcal{M}+1}\sum_{\kappa=0}^{\mathcal{M}} \underbrace{\boldsymbol{W}\,\boldsymbol{A}(\boldsymbol{\omega_\kappa})\,\boldsymbol{V}}\; c_i(\omega_\kappa) \qquad q_{i,red} = \frac{2}{\mathcal{M}+1}\sum_{\kappa=0}^{\mathcal{M}} \underbrace{\boldsymbol{W}\,\boldsymbol{b}(\boldsymbol{\omega_\kappa})}\; c_i(\omega_\kappa)$$

$$\Rightarrow \left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega)\,\boldsymbol{T_{i,red}}\right) x(\omega) = \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega)\,\boldsymbol{q_{i,red}}$$

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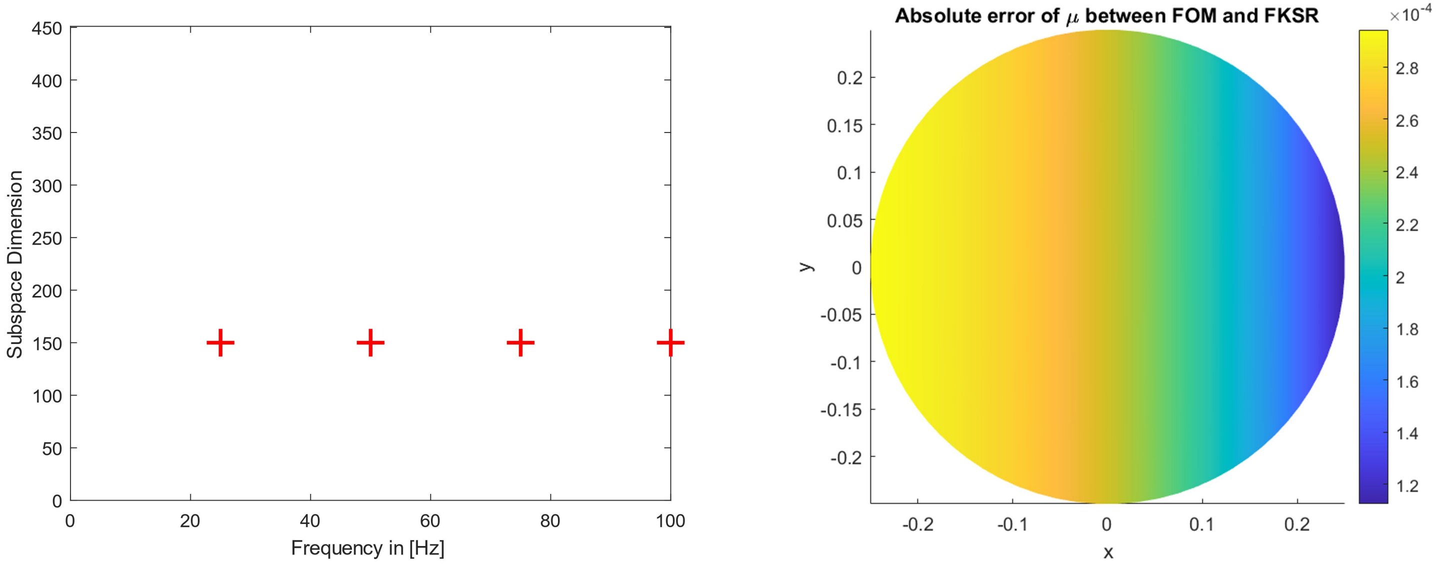
Fixed Krylov Subspace Recycling (FKSR)

Drawback of FKSR:

- Krylov subspace dimension m and number of sampling master frequencies Ω are **predetermined by the user!**



(a)

(b)

Figure 7.: (a) Subspace dimension of Krylov subspaces employed at Ω for the construction of the reduction bases employing different sampling patterns, (b) Absolute error of μ between the FOM and FKSR

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Model-Order Reduction within IGA-framework

Introduction of Automatic Krylov Subspace Recycling (AKR)

Automatic Krylov Subspace Recycling (AKR) [Pa21]

- Motivation: Find optimal settings to construct reduction basis
- Adaptive procedure that allows order m and No. of sampled master frequencies in Ω to vary with frequency and produce a ROM that is under a predefined error threshold
- Dimension $m(\omega)$ of the respective Krylov subspace $\mathcal{K}^{\omega}_{m(\omega)}$ for each $\omega \in \Omega$ as well as Ω are determined iteratively through an automated procedure
- In each iteration a residual is built and compared to a user-defined error threshold

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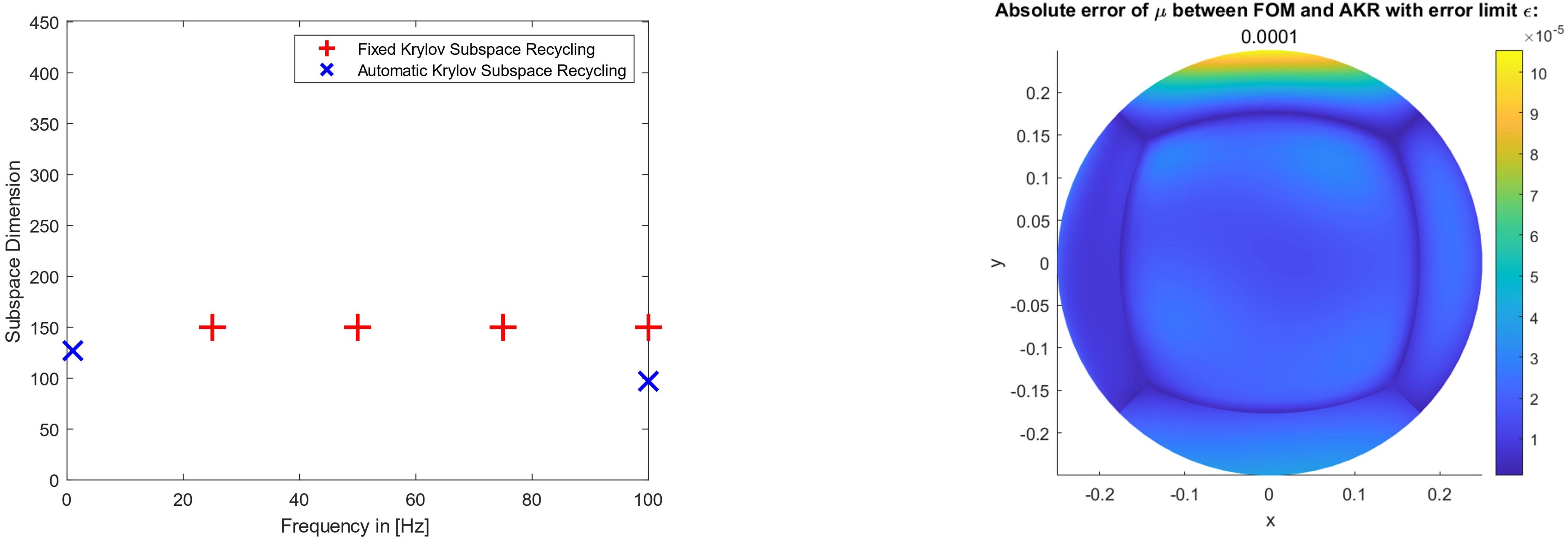
Automatic Krylov Subspace Recycling (AKR)



(a) (b)

Figure 8.: (a) Subspace dimension of Krylov subspaces employed at Ω for the construction of the reduction bases employing different sampling patterns, (b) Absolute error of μ between the FOM and AKR with error limit $\epsilon = 1\mathrm{E}-04$

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Future work

Acoustics and vibro-acoustics

- Extend MoR scheme for BEM systems for multiple design variables for shape optimization

So far:

- Only considered the acoustic domain

Future work vibro-acoustic:

- Just coupling at the interface (boundary) of fluid domain and structural domain
- Assume FEM for structure and BEM for acoustic domain
- Explore MoR schemes for fully coupled vibro-acoustic systems

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Thank you!

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Date: 01/09/2024

References

Acoustics and vibro-acoustics

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