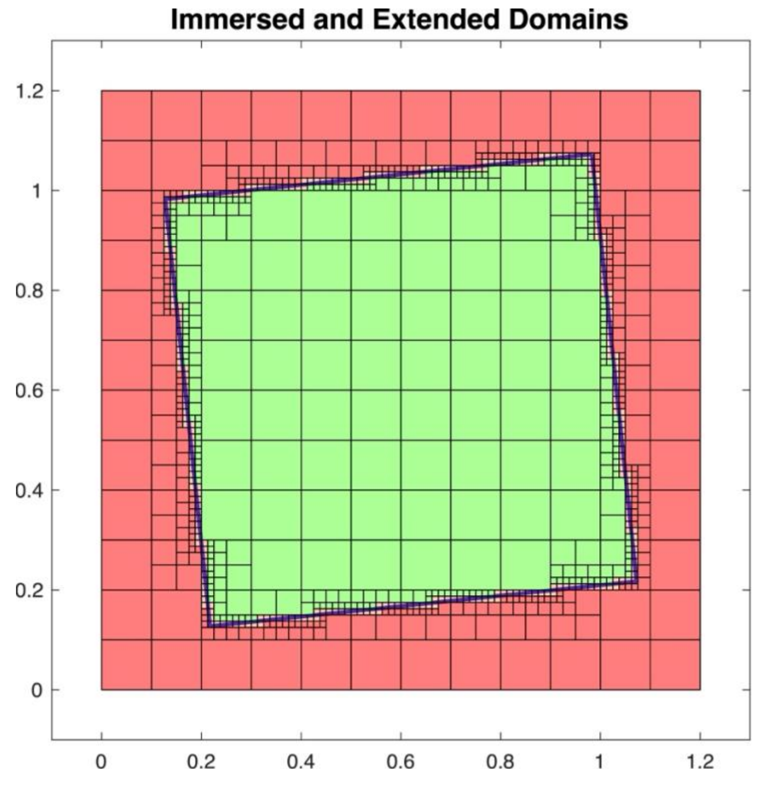
Gecko Technical Report 1

DC7 – Angelos Pagonas



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Executive summary

This report outlines progress in advancing Isogeometric Analysis (IGA) within the immersed framework, focusing on its potential for solving dynamic problems. The work contributes to the objectives of the GECKO project under the MSCA network. It addresses key challenges regarding computational efficiency and accuracy, while fostering innovation in numerical simulation methodologies.

After a brief introduction on the research background, some prerequisites and established research is presented, as well as a quick overview of the doctoral candidate's learning path. Especially, emphasis is given on existing results that were recreated and explain the motivation of the research. On a simple immersed bar problem there is an analysis of trade-offs between computational efficiency and accuracy using consistent and lumped mass matrices, as well as α-stabilization techniques. Later the methodology of the extended studies is presented. It includes novel research in bar and beam with harmonic excitation test examples for wave propagation accuracy studies. Additionally, extension of the work in 2D, including immersed square with its axis aligned or rotated with respect to the extended domain axis, in the latter case using a different approach by introducing the quadtree-based integration schemes. The available novel results for 1D and 2D domains are demonstrated and discussed.

This research underscores the potential of immersed IGA for dynamic simulations, demonstrating advancements in stability and efficiency while identifying areas for further research. Challenges, such as optimizing accuracy in higher-order problems and enhancing quadtree implementations for broader applications, remain key areas for investigation. For the near future, work will focus on completing studies for rotated domains and possibly introducing another quadtree example, and in the meantime addresing unresolved issues in accuracy analyses for beams and plates.

This report marks substantial progress in immersed IGA methodologies, establishing a solid foundation for further innovation and practical applications.

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CAD	*Computer Aided Design*
FEM	*Finite Element Method*
IGA	*IsoGeometric Analysis*
PDE	*Partial Differential Equation*

Introduction

This deliverable documents the progress achieved in research focused on immersed Isogeometric Analysis (IGA) for structural dynamics. The long-term aim is to address key challenges in the field such as the accuracy-efficiency dilemma, but at first the focus is on performing case studies on 1D and 2D immersed structural elements.

The objectives of the deliverable include:

- Establishing a research foundation
- Developing novel methodologies
- Presenting results

This report is organised into five main chapters:

1. Research background and learning path
2. Methodology
3. Results
4. Work in progress and future work
5. Conclusions

This deliverable not only demonstrates the technical progress achieved but also underscores the academic and research development, which are fundamental goals of the MSCA network.

1 RESEARCH BACKGROUND AND LEARNING PATH

This chapter briefly reviews key advancements in IGA, focusing on its evolution from basic principles to applications in Dynamics and ultimately Immersed Dynamics and Mass Lumping, and follows a parallel approach to the Doctoral Candidate's personal incremental learning process, where foundational results were studied, recreated and extended step by step to the existing state of the art.

This approach not only provides a background and sets the stage for the novel contributions presented in the next section but also demonstrates the step-by-step process that prepared the foundation required to achieve them, which indeed constitutes the first period of the research progress. This dual focus not only contextualizes the presented work for the reader but also demonstrates the problem-solving and skill-building that align with the goals of the MSCA network, fostering academic growth and research training.

1.1 IGA Advantages

Isogeometric analysis can provide numerous advantages compared to conventional Finite Element Analysis (FEA). Originally IGA was conceived as an idea of integrating Computer Aided Design (CAD) and FEA, providing efficiency by eliminating the meshing and re-meshing processes to significantly reduce the preparation time for simulations. Another strong motivation in its initial conception was the reduction or elimination of errors due to geometric approximation. [Hu05]

These reasons were already compelling enough for the development of IGA, nevertheless even more benefits emerged afterwards, like the effect of higher-order inter-element continuity due to the smoothness of its basis functions, enabling superior approximation properties. The benefits of improved spectral accuracy over classical finite elements analysis (fig. 1) established IGA as a strong high-order competitor in the field of structural dynamics. [Co06]

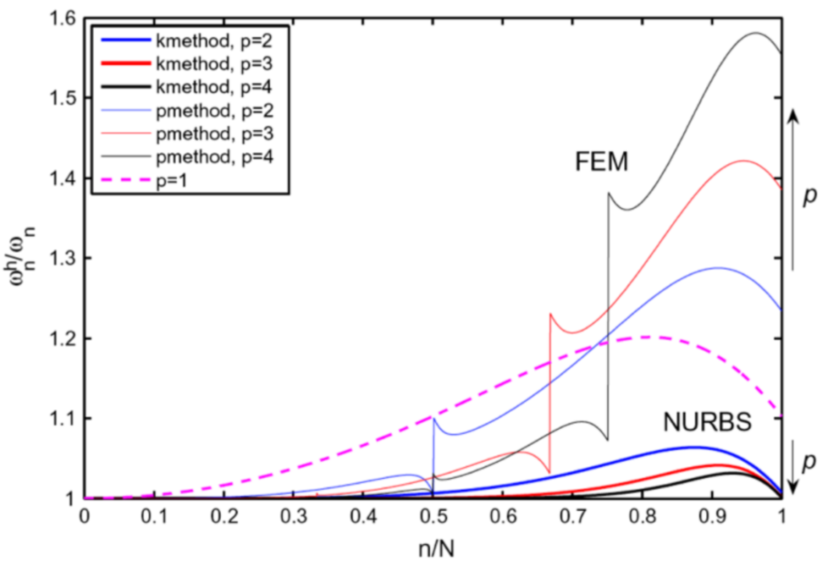

Figure 1: Improved spectral accuracy of IGA (NURBS – k-method) compared to FEM (p-method).

1.2 Immersed IGA for Dynamics

The maximum eigenfrequency has a major role in explicit dynamics methods, since it is dictating the critical time step that ensures stability for the simulation. In immersed methods the presence of small cut elements can produce very large maximum eigenfrequencies, resulting in infinitesimal time steps to ensure stability, and thus to infeasible simulation times.

1.2.1 Advantages in efficiency and stability

Immersed IGA is an alternative to standard FEA that could provide stability and efficiency, leveraging the effect of higher-order inter-element continuity in conjunction with either a lumped mass matrix or by introducing α-stabilization in the context of Finite Cell Method, along with a Consistent Mass Matrix.

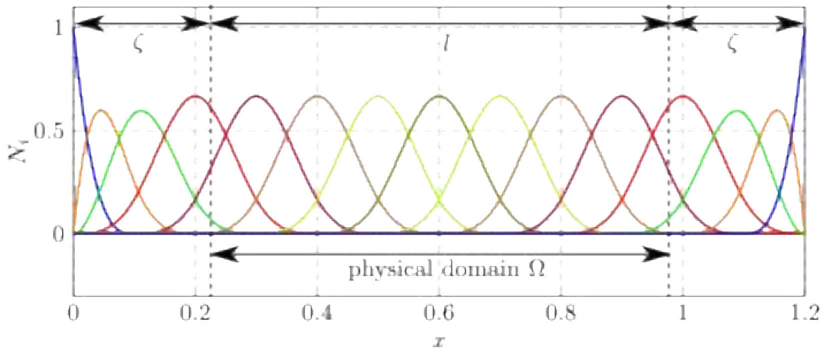

Figure 2: IGA problem setup with B-spline basis functions for an immersed 1D structural element (bar/beam) of length l embedded in an extended domain [0, L] with fictitious domain size ζ on each side.

As shown originally in [Le20], Isogeometric B-spline (IGA) discretizations in an immersed framework and in combination with mass lumping can lead to a greatly reduced largest eigenvalue (see **fig. 3**), while for standard FEM based on Lagrange polynomials the largest eigenvalue diverges as the support of an element approaches zero.

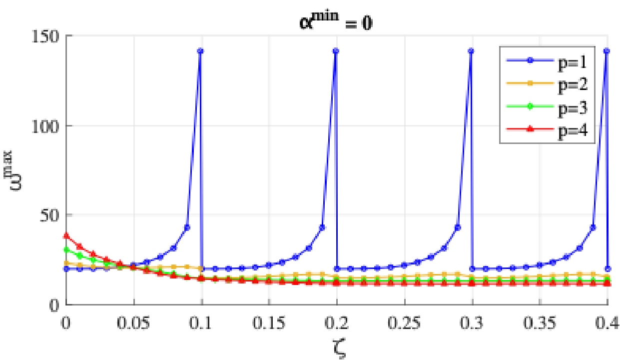

Figure 3: Restricted largest eigenvalues (for p>1) independently of the fictitious domain size ζ in a bar, for computations with a lumped mass matrix.

For immersed IGA discretization, when consistent mass matrix is utilized and lumping is avoided, the largest eigenfrequencies for different immersion configurations follow the trend as in the standard FEA where they diverge to infinity as the support of the element becomes smaller (see **fig. 4** left). In [Ra24] it is demonstrated that if a consistent mass matrix is used along with the implementation of α-stabilization, then in the immersed bar example the largest eigenvalue is also bounded (see **fig. 4** right), although in smaller proportion compared to the Lumped Mass Matrix case.

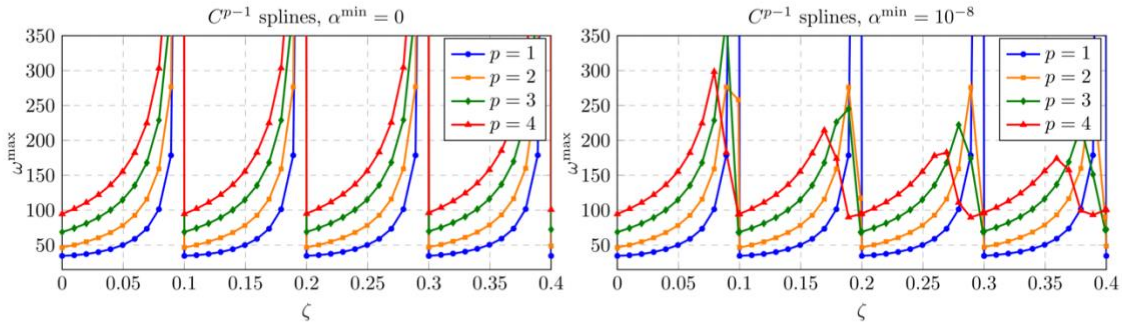

Figure 4: Largest eigenvalue over fictitious domain size ζ in a bar, for computations without stabilization (left) and with stabilization (right) (bounding of the eigenvalues) and a consistent mass matrix.

1.2.2 Limitations in Accuracy

While the bounding of the maximum eigenfrequency due to mass lumping has a very positive effect in the efficiency of the simulation allowing larger time steps, on the other hand it introduces limitations in accuracy, which depending on the application could be crucial.

This time, the quantity under review is the period error, since in the harmonic vibration the structure is supposed to return in its exact position after a period. Hence by measuring this L2 error we have an indication of the accuracy of the current implementation, and we do that for several immersed configurations as well as the boundary fitted as an extreme case (ζ=0), in a similar way we performed the largest eigenvalue study earlier.

In **figs. 5, 6**, the results of the L2 error after a period, along with the critical time step of a gaussian pulse wave propagation problem are presented in the same bar setup that we used in the previous chapter to demonstrate the improvement in efficiency and stability.

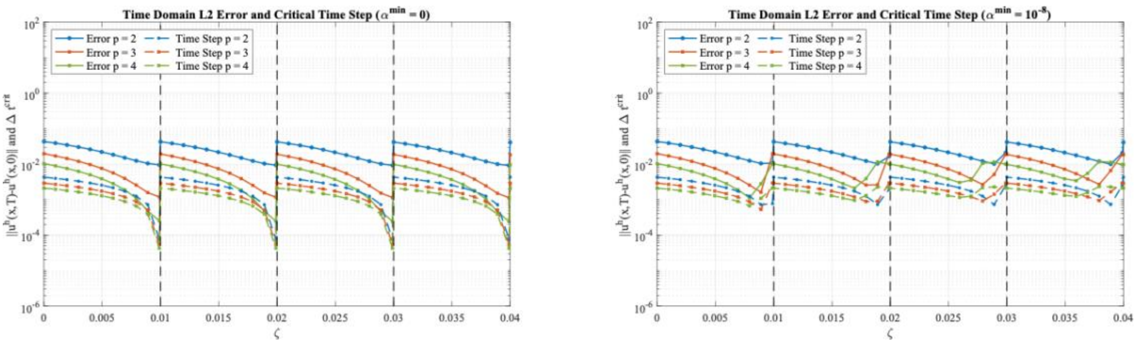

Figure 5: Error after a period and critical time step for a gaussian pulse wave propagation problem in a bar with a consistent mass matrix.

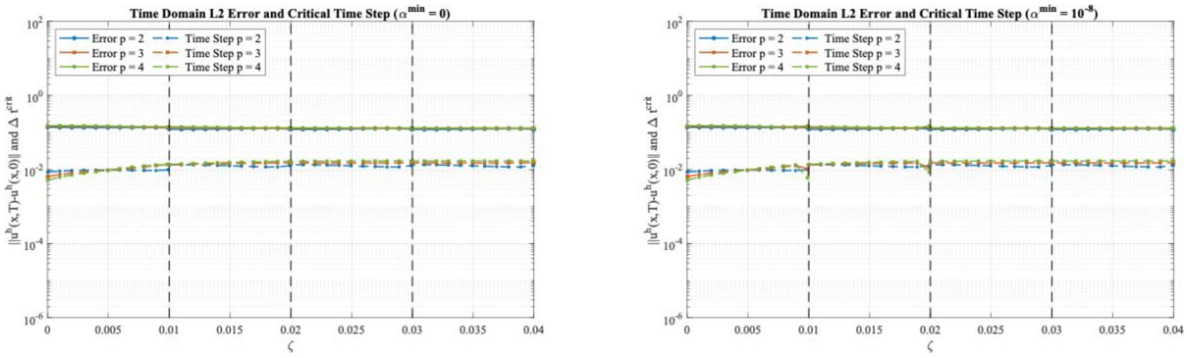

Figure 6: Error after a period and critical time step for a gaussian pulse wave propagation problem in a bar with a lumped mass matrix.

We can observe that although the critical time step is increased by transitioning to a lumped mass matrix (**fig. 6**) allowing faster simulation times, there is the negative consequence of the error increasing significantly, leading to results with limited accuracy compared to equivalent results obtained with a consistent mass matrix (**fig. 5**).

1.3 Learning *path and results recreation*

Studying and implementing IGA problems started in a similar sequence as presented above, from the basic and general to the specific and state of the art. The first steps emphasized on fundamental understanding of IGA and implementation of corresponding problem solving through coding, later the focus was on structural vibrations and so results from [Co06] where also recreated.

Finally, moving to the immersed IGA approach for structural dynamics and mass lumping effects or stabilization along with a consistent mass matrix which are demonstrated above. After getting an understanding of those matters, doctoral candidate managed to create algorithmic implementations that reproduced the results of [Le20] and [Ra24] and are presented in the previous pages. That process was the optimal way to engage progressively from the fundamentals to the advanced, building up specific knowledge and skills. The next step was to extend the promising studies of [Ra24] and produce novel research results.

2 METHODOLOGY

2.1 Bar

The accuracy studies performed in [Ra24] employed a wave propagation example of a gaussian pulse (as presented in **fig. 7**) while the additional novel equivalent studies a harmonic propagation (**fig. 8**).

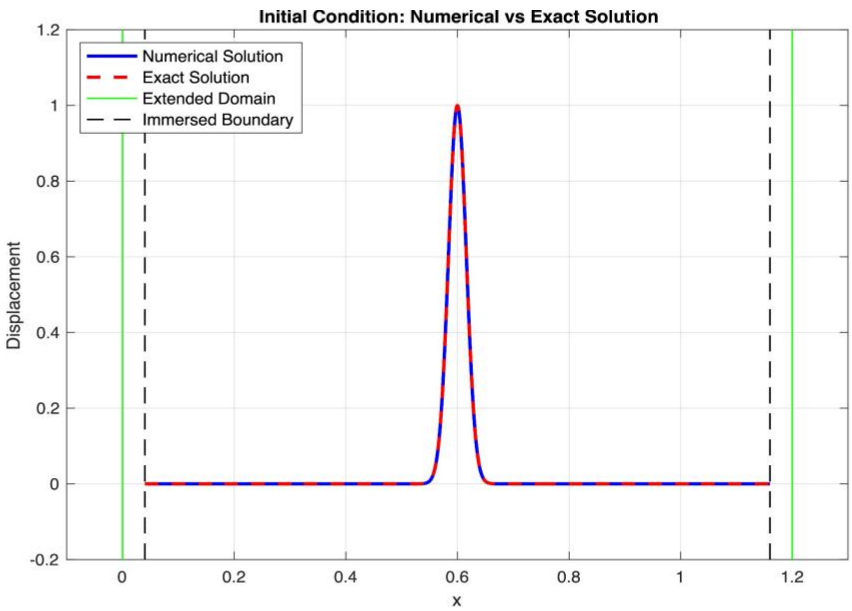

Figure 7: Gaussian pulse initial condition for wave propagation problem in immersed bar

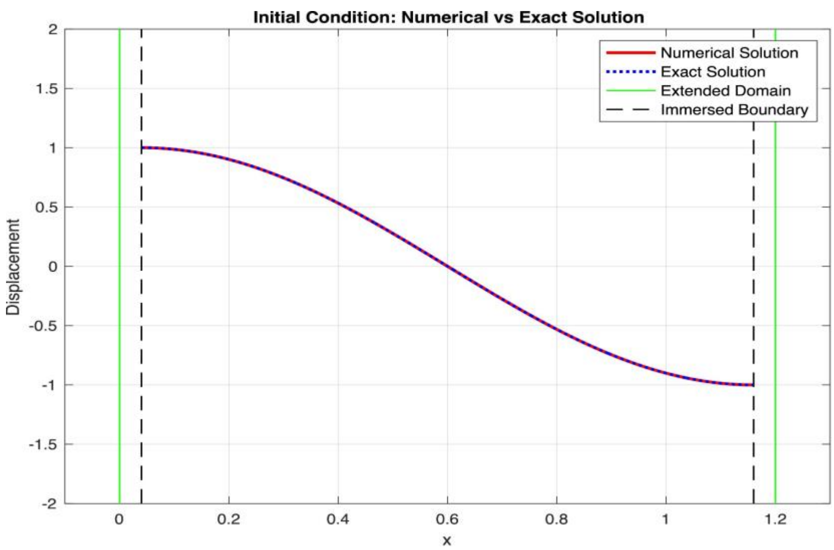

Figure 8: Harmonic vibration initial condition for wave propagation problem in immersed bar

2.2 Bernoulli – Euler beam

The current case study is examining the effects of transitioning from 2nd to 4th order Partial Differential Equation (PDE). The code implementation and problem setup are similar to the bar problem as presented in chapter 1. The harmonic propagation initial condition can be seen in **fig. 9**.

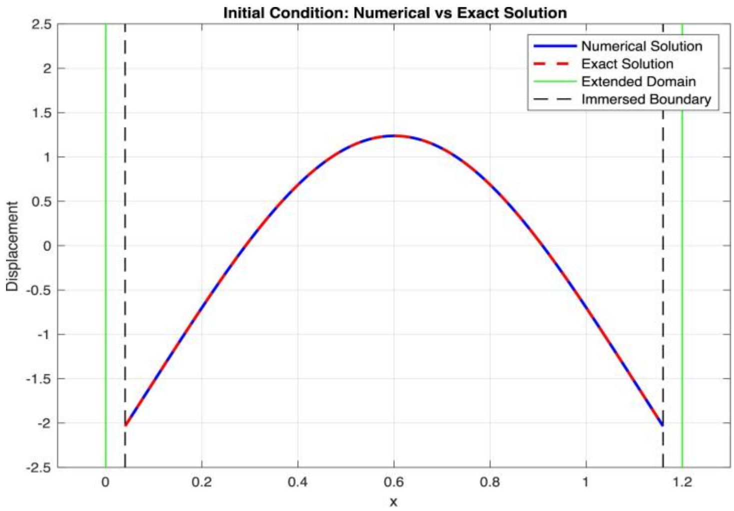

Figure 9: Harmonic vibration initial condition for wave propagation problem in immersed beam

2.3 Extension to 2D (Membrane and Poisson - Kirchhoff Plate)

2.3.1 Immersed square with aligned axis to the extended domain

The next step in this research direction was the extension of the 1D results to 2D square elements. As a first approach the new problem setup is very similar to the simpler one-dimensional since the immersed and extended domains have their axis aligned (**fig. 10**). In this way, the integration of cut elements can be exact by splitting them on the domain border into 4 sub elements and integrating only those inside the immersed domain, following the same approach implemented in the one-dimensional domain where each cut element is split in two at the exact immersion border.

Specifically, the equivalent of the bar is the membrane (2nd order PDE) and accordingly for the Bernoulli – Euler beam the Poisson – Kirchhoff plate (4th order PDE).

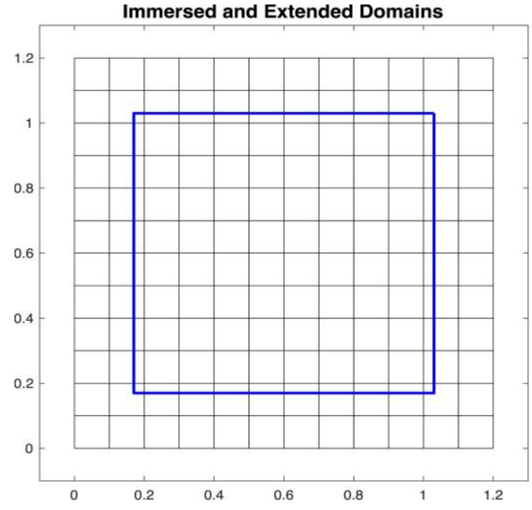

Figure 10: Extension to 2D square problems with axis aligned between immersed and extended domains

To perform the accuracy studies a wave propagation example with harmonic excitation initial conditions was utilized (**fig. 11**).

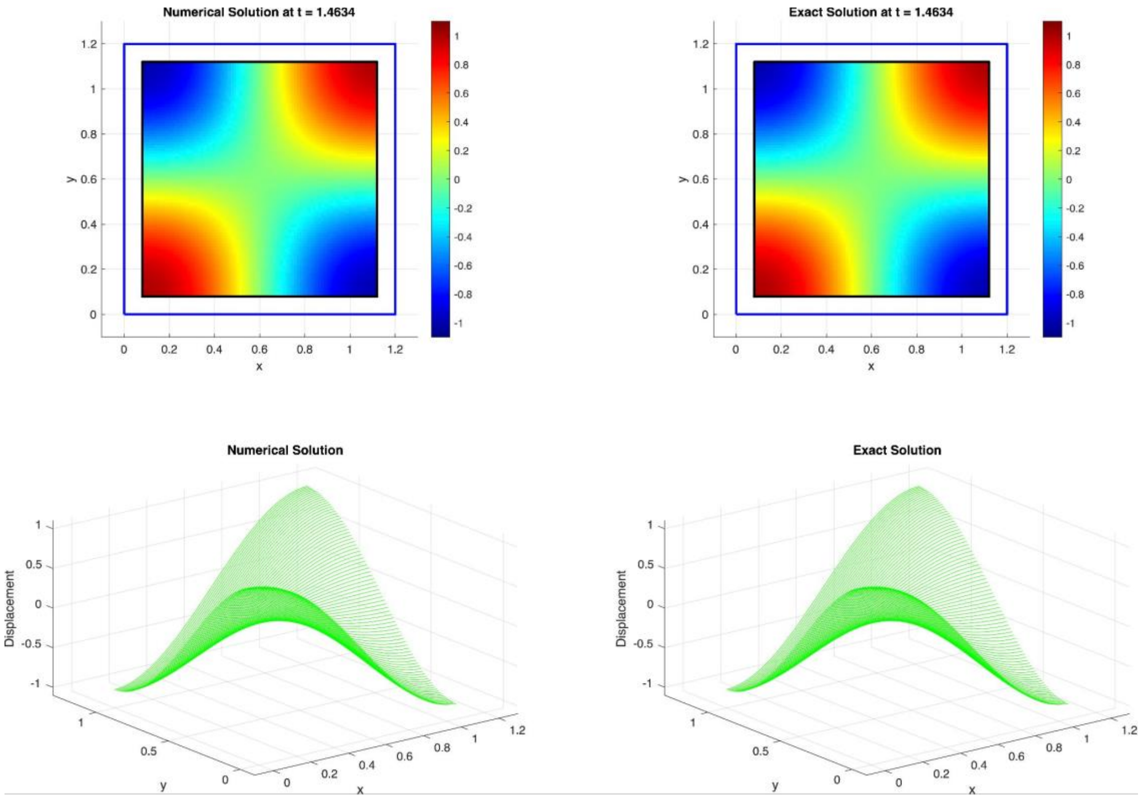

Figure 11: Harmonic vibration initial condition for wave propagation problem in a squared membrane with axis aligned between immersed and extended domains

2.3.2 Immersed square problem with rotated axis to the extended domain

A following extension that helps to generalise more the examples is the modification of the immersed square by rotating it, in this way the immersed and extended domains don't have their axis aligned anymore (**fig. 12**), which changes significantly the process of the overall numerical integration implementation.

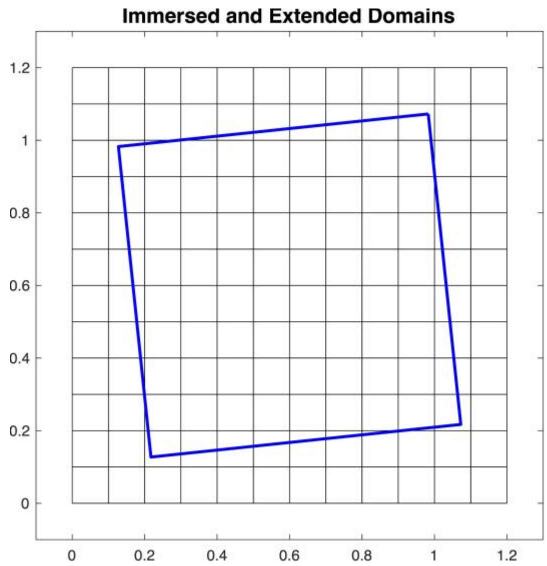

Figure 12: Extension to 2D square problems with rotated axis between immersed and extended domains.

The quadtree scheme is employed to tackle that challenge as a first approach (**fig 13**). This implementation could potentially deal with any 2D shape ensuring there is an algorithm providing the information of points being inside or outside the immersed domain.

Each element is divided in four equal sub-elements, and this can be selectively repeated in each (sub)-element for several iterations. Integration takes places according to the indicator value assigned accordingly either as 1 or alpha (alpha equal to either zero or a small tolerance value for stabilization), and in principle following the concept of the Finite Cell Method as previously.

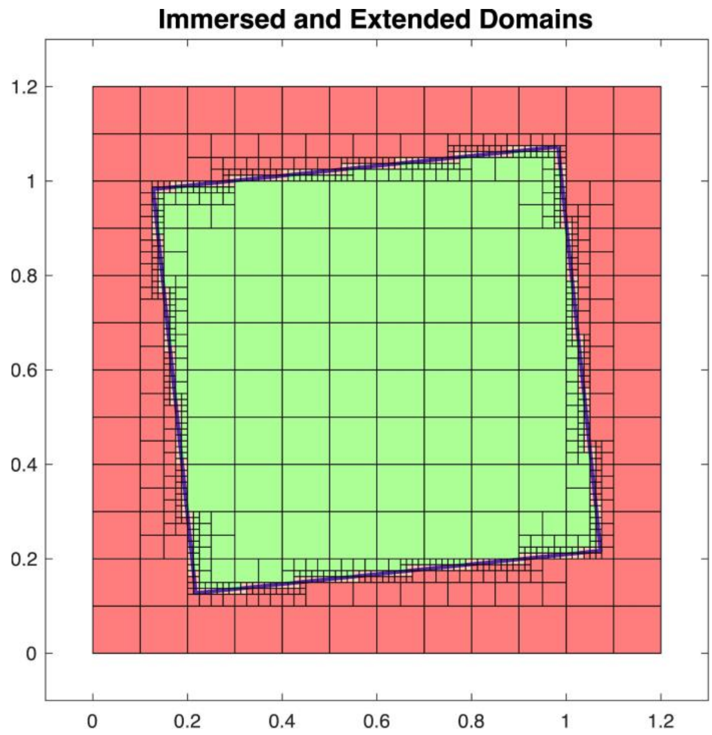

Figure 13: Implementation of a quadtree integration approach for the rotated square problem

When the maximum level of iterations is reached then the smallest sub-element can be integrated per Gauss point according to their position (inside or outside the immersed domain) as is showcased in **fig. 14**.

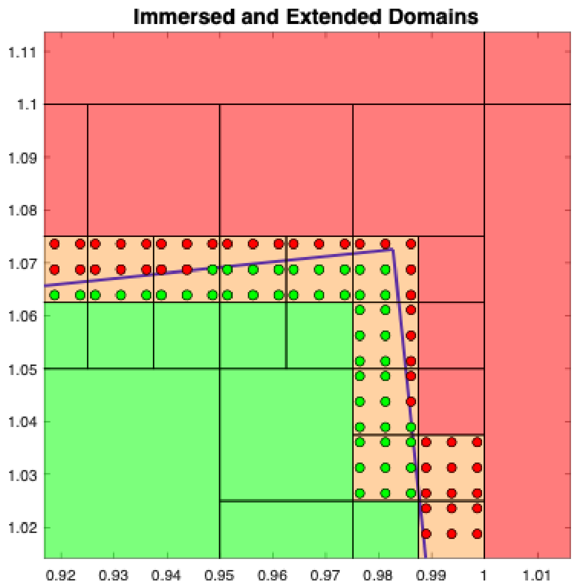

Figure 14: Detail of quadtree quadrature integration per Gauss point in cut elements

3 RESULTS

In this chapter the originally created results are introduced, providing an extension of those displayed in the first chapter. As a general comment, the behaviour follows the same trend as the results presented in chapter 1, although there may be some small differentiations and additionally some results not yet interpreted completely.

3.1 1D Domain

3.1.1 Bar

Following the same approach to the recreated results introduced previously, by quantifying the L2 error after a wave propagation period, we can examine the accuracy, since the structure is supposed to return in its exact position after a period in the chosen examples. The results obtained by employing a harmonic vibration for the bar are presented below.

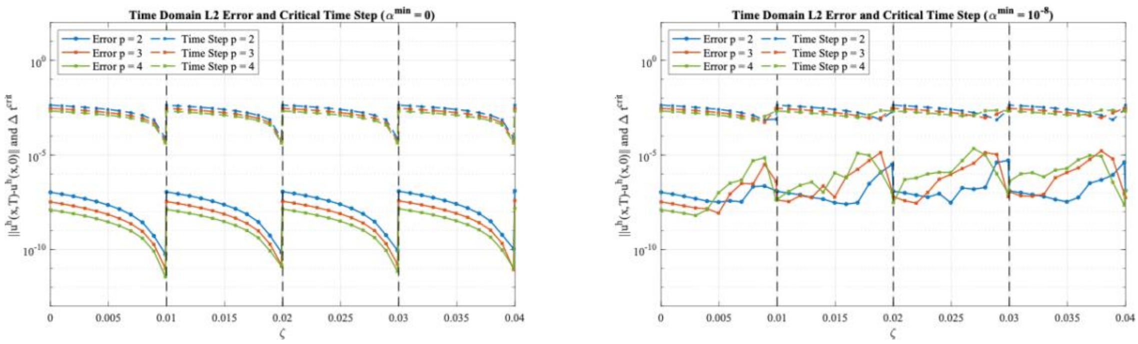

Figure 15: Error after a period and critical time step for a harmonic wave propagation problem in a bar with a consistent mass matrix

Again, we observe the same trend compared to the gaussian pulse results above and the same transformation when transitioning to a lumped mass matrix.

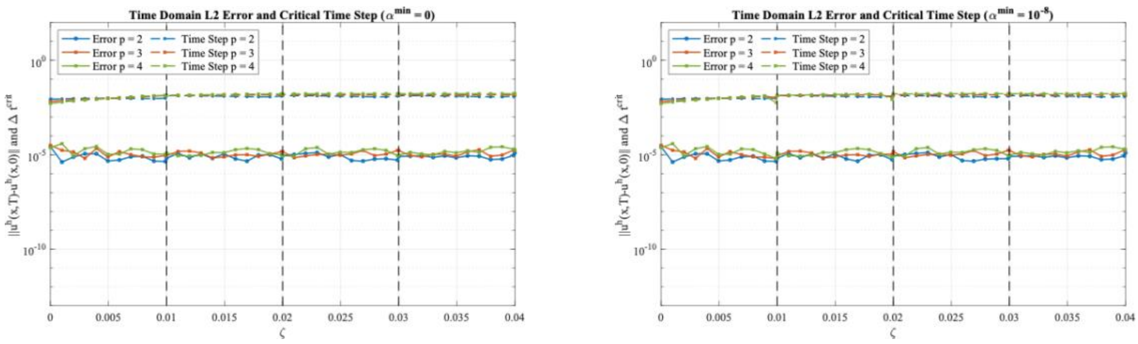

Figure 16: Error after a period and critical time step for a harmonic wave propagation problem in a bar with a lumped mass matrix.

Interestingly, we observe that for this harmonic vibration example, the effects of α-stabilization diminish significantly the accuracy.

3.1.2 Bernoulli – Euler beam

An extension of the bar results to 4th order partial differential equation problems was presented in [Pa24] employing an immersed beam setup with a harmonic excitation wave propagation example, as presented in the previous chapter.

We can observe the effects of α-stabilization in conjunction to a consistent mass or the introduction of the mass lumping to the maximum eigenvalues for several immersion scenarios.

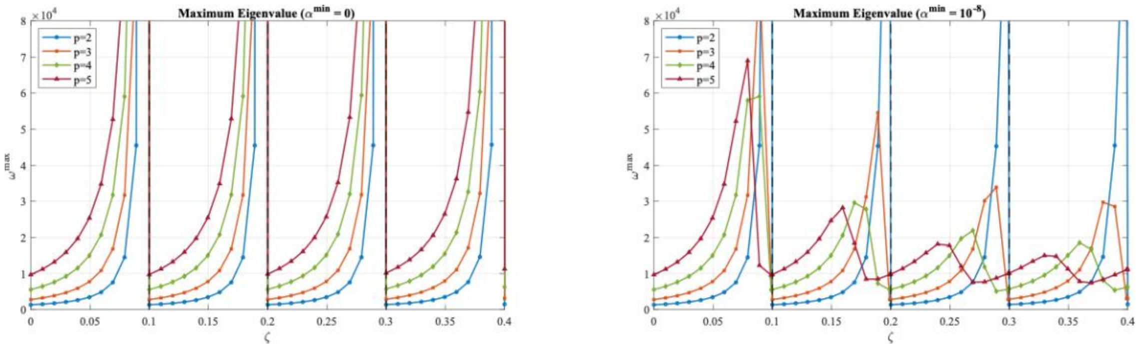

Figure 17: Largest eigenvalue over fictitious domain size ζ in a beam, for computations without stabilization (left) and with stabilization (right) (bounding of the eigenvalues) and a consistent mass matrix.

The bounding behavior is present and mimics the effects observed in the bar problem in both cases.

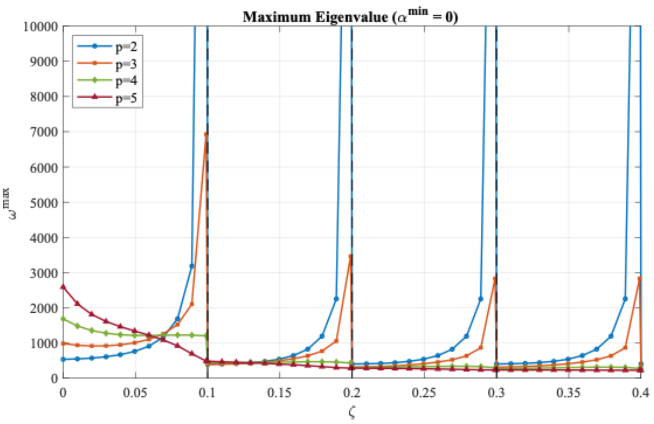

Figure 18: Reduced largest eigenvalue (for p>3) independently of the fictitious domain size ζ in a beam, for computations with a lumped mass matrix.

Regarding the accuracy study with consistent mass (fig. 19), the results obtained are not exactly the expected and there is still some pending work in progress to comprehend them.

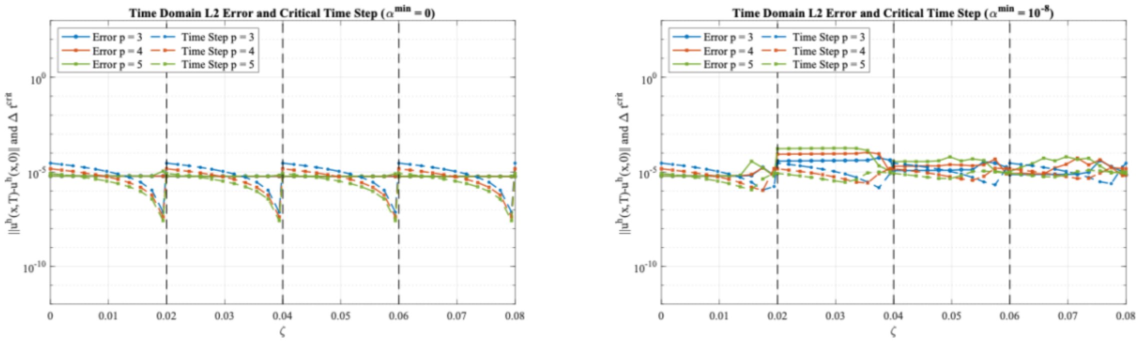

Figure 19: Error after a period and critical time step for a harmonic wave propagation problem in a beam with a consistent mass matrix

As we can observe the error for the consistent mass matrix case is not following the time step trajectory as already presented in the bar example, or later will be presented for the membrane study.

On the contrary, lumped mass results seem to behave in a more predictable manner.

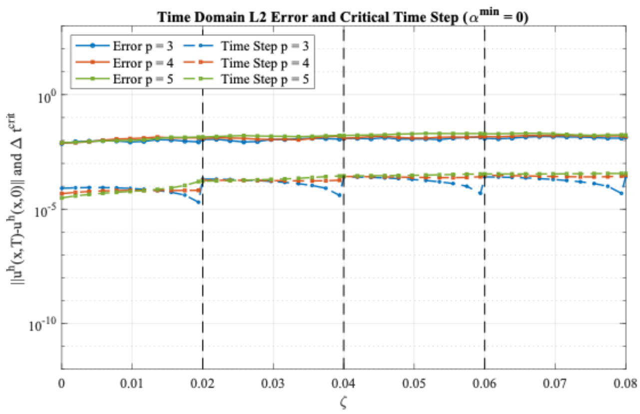

Figure 20: Error after a period and critical time step for a harmonic wave propagation problem in a bar with a lumped mass matrix.

3.2 2D Domain

As discussed previously a research goal of the current PhD work is to extend and further enrich the 1D studies to their equivalent to 2D example problems. Albeit there is work in progress, the completed studies are demonstrated below. The available findings are limited to the aligned axis test case, and for the Poisson-Kirchhoff plate are limited only to the largest eigenvalue study.

3.2.1 Membrane

The results of the membrane are matching the bar results as we can observe for the largest eigenvalue and wave propagation accuracy studies following.

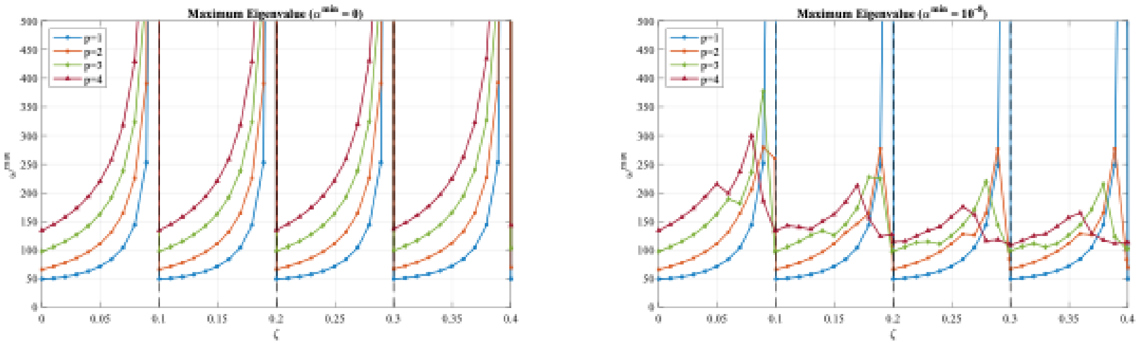

Figure 21: Largest eigenvalue over fictitious domain size ζ in a membrane, for computations without stabilization (left) and with stabilization (right) (bounding of the eigenvalues) and a consistent mass matrix.

The bounding of the maximum eigenfrequency is taking place with the introduction of α-stabilization (fig. 21) as well as with mass lumping in a more prominent way (fig. 22).

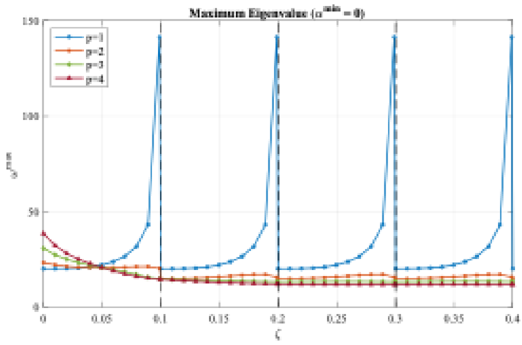

Figure 22: Reduced largest eigenvalue (for p>1) independently of the fictitious domain size ζ in a membrane, for computations with a lumped mass matrix.

The results of the accuracy study employing the wave propagation with harmonic excitation example are shown in fig. 23 and 24, utilizing a consistent and lumped mass matrix accordingly.

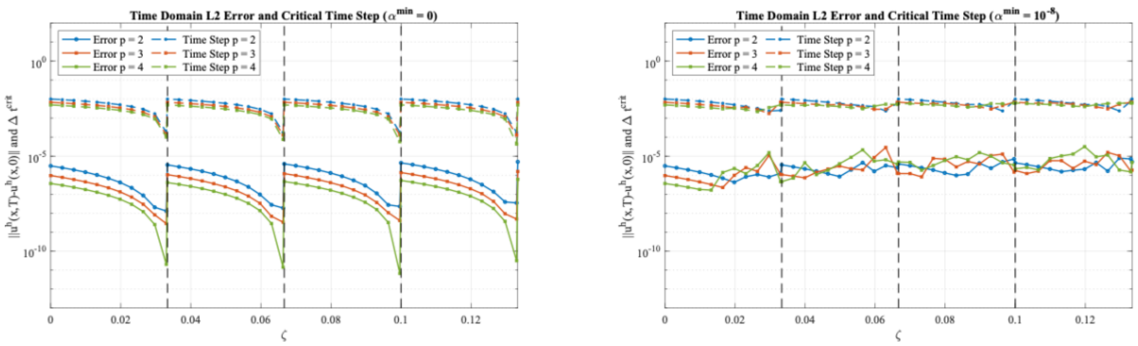

Figure 23: Error after a period and critical time step for a harmonic wave propagation problem in a membrane with a consistent mass matrix.

The findings are comparable to those of the bar applying the harmonic vibration example, and additionally we can comment that the reduction of accuracy due to lumping is more prominent.

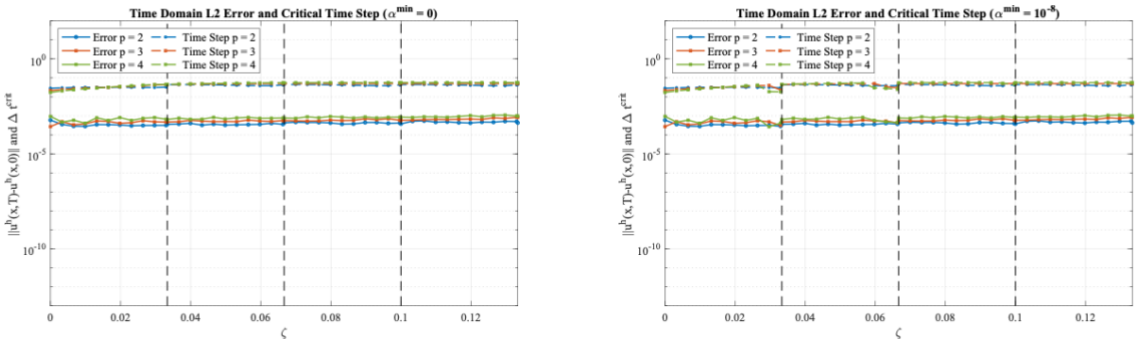

Figure 24: Error after a period and critical time step for a harmonic wave propagation problem in a membrane with a lumped mass matrix.

3.2.2 Poisson – Kirchhoff plate

The current element represents the equivalent of the Bernoulli – Euler beam in two dimensions. Both the consistent and lumped mass versions of the plate show great similarities to the performance of the maximum eigenfrequencies studies, along with their bounding as shown already for the beam.

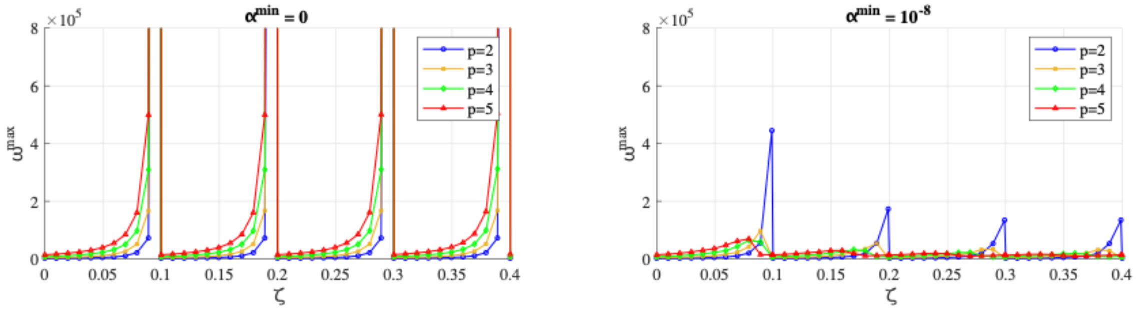

Figure 25: Largest eigenvalue over fictitious domain size ζ in a Poisson - Kirchhoff plate, for computations without stabilization (left) and with stabilization (right) (bounding of the eigenvalues) and a consistent mass matrix.

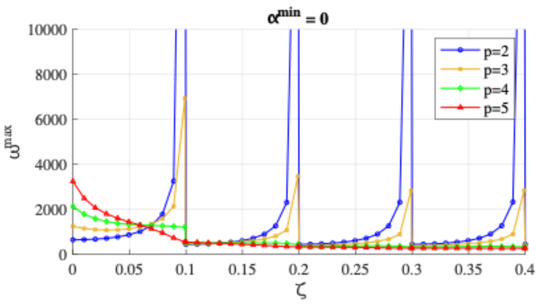

Figure 26: Reduced largest eigenvalue (for p>3) independently of the fictitious domain size ζ in a Poisson - Kirchhoff plate, for computations with a lumped mass matrix.

4 WORK IN PROGRESS AND FUTURE WORK

Currently research activity is focused on completing the studies for the aligned axis square domains likewise for the plate and experiment with several setups in the rotated immersed square membrane and plate.

The transition from the immersed square with aligned axis to a new approach where the quadtree scheme is employed is a step forward that guarantees some continuity and relevance to the previous results and an attempt to benchmark the new approach with the existing simpler (aligned axis) implementation. In this way the new more complex approach can be gradually implemented and compared, employing the quadtree approach to solve a test case including a non-rotated square, since this example is also solved with the simpler implementation.

Another example could be also employed, for example a circular 2D domain instead of a square for which analytic solutions are also available.

The same approach and examples could be implemented for the plate after the membrane results are complete, and the methodology works. Still the accuracy results of the Bernoulli – Euler beam need to be explained or the methodology to be inspected again for errors, since the current findings are not yet interpreted.

After the conclusion and summing up of all these findings, hopefully a journal publication could be composed, along with dissemination of the results to several conferences and events.

5 CONCLUSIONS

This technical report presents a comprehensive overview of the research progress, methodologies, as well as results achieved during the first period of this doctoral work.

Some key achivements include:

1. Establishing a strong understanding of IGA principles, validated and enriched through recreation of foundational results, as well as extension to immersed approach.
2. Quantifying accuracy and stability trade-offs in dynamics simulations, utilizing wave propagation test cases with both Gaussian pulse and harmonic excitation initial conditions.
3. Extending studies to the 2D domain as well as 4th order PDEs**,** along with more advanced configurations such as the the quadtree implementation with potential to deal with complex geometries.

Some of the encountered challenges are actually the open topics currently and relate to the accuracy limitations introduced by mass lumping, the interpretation of unexpected findings in the beam accuracy studies and the wave propagation problem setup in plate.

The plan for future work is to resolve the issues and completing the 2D studies, as well as prepare findings for journal publication.

The current work has established a robust foundation for extending the state of the art in immersed IGA for dynamics and the next steps aim to redifine it by adressing unresolved challenges.

6 REFERENCES

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