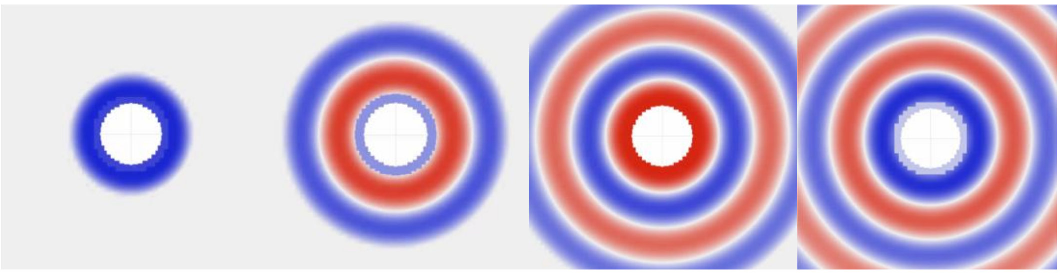
Gecko
Design for IGA-type discretization workflows

# Gecko Technical Report 1

# DC8 – Doğuhan Kılıçarslan



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# Executive summary

This technical report is compromised of the research conducted about transient acoustics with the implementation and verification of the time domain boundary element method, as well as the future plans to progress in the research.

# Table of contents

## List of figures

## List of abbreviations

| | |
|---|---|
| *IGA* | *IsoGeometric Analysis* |
| *FEM* | *Finite Element Method* |
| *FEA* | *Finite Element Analysis* |
| *CAD* | *Computer Aided Design* |
| *NURBS* | *Non-Uniform Rational B-Splines* |
| *BEM* | *Boundary Element Method* |
| *MOR* | *Model Order Reduction* |

# Introduction

Even though acoustics is mostly related with frequency domain problems and solution techniques there are still many problems that require transient solutions, such as; Auralization problem where the time of arrival of a sound to both ears are different and transient solution is required to understand the human hearing [1], or, pass-by-noise problem [2] where the moving cars creates traffic noises which generally requires transient modelling of the problem. There are also problems that the input signal is directly generated by transient sound sources that carry frequency signals changing in time such as engine run-up noise [3]. Since these simulations are conducted through time domain analyses or convolution integrals, computational requirements are very high alongside the fine meshes required to capture the detailed and/or very detailed geometries. On top of that, many of the aforementioned problems require remeshing due to changing conditions or to take care of the uncertainties which increases the computational burden for the simulation. Where the computational cost and accuracy errors may become large enough, so that some problems may be better tackled with experimental tests instead of numerical simulations.

Isogeometric Analysis (IGA), proposed by Hughes et. al. [4], tries to combine computer aided design (CAD) with finite element analysis (FEA), and, while doing so offers a robust solution to combination of problems mentioned. By utilizing CAD geometry descriptions, such as non-uniform rational B-splines (NURBS), as FEA shape functions, geometry is exactly represented which eliminates any discretization error and reduces the “fineness” of the mesh such that the original size of the system is reduced without losing details. Moreover, since the changes in the geometry is reflected directly to FEA description, need for meshing (ergo remeshing) is eliminated. Specially for time domain analysis, IGA also enables the use of similarly sized elements without losing the geometric details or using many elements with less detailed regions just to increase or preserve the stability of the problem. Finally, since NURBS has great flexibility with relatively low number of parameters it is also very efficient for topology optimization problems [5] that is placed in between CAD and FEA such that IGA is very beneficial to bridge that gap as well [6]

However, as many solutions in engineering IGA also comes with some new or exasperated older problems. One such problem is that finite element method (FEM) requires volume meshes in 3 dimensions and surface meshes in 2 dimensions while CAD representations are generally surfaces and lines for 3 and 2 dimensions, respectively. Hence, special methods may become necessary to apply IGA efficiently to complex and trimmed geometries such cut FEM [7], isogeometric boundary representation analysis [8], or shifted boundary method [9] . Alternative is to not use FEM, but instead to use boundary element method (BEM) in which only boundary is required such that it matches with CAD representations [10] even though it comes with its own problems such as non-conforming patches present in general CAD drawings [11].

In this report application of IGA workflows to transient acoustic problems are investigated and their implementations and developments are shown.

# 1. Transient acoustics BEM

## 1.1 Theory

Governing differential equation for transient acoustics is known as acoustic wave equation and is written as,

$$c^2\nabla^2 p(r,t) - \frac{1}{c^2}\frac{\partial^2 p(r,t)}{\partial t^2} = f(r,t), \qquad in\Omega$$

where $p$ is the pressure, $c$ is the speed of sound, $f$ is a general acoustic excitation. Here $\nabla^2$ is the Laplacian operator defined as $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in cartesian coordinates.

BEM starts by defining a representation formula in which function values inside the domain $\Omega$ can be represented from the function values on the boundary $\Gamma$ of the same domain [12]. For a static case (for simplicity) equation can be written as,

$$p(r) = \int_\Gamma \big(p^*(\xi,r)\nabla p(\xi) - \nabla p^*(\xi,r)p(\xi)\big)d\Gamma,$$

where $p^*$ is the fundamental solution that is defined by the response of a point source in an infinite domain. If the point to represent is moved to the boundary in a limiting process same equation can be used to solve for the unknown boundary values of the function.

For the time domain analysis same logic applies with a change of time response of an impulsive point source applied in an infinite domain and an additional time integral given as,

$$p(\xi,t) = \int_0^t \int_\Gamma \big(p^*(\xi,\tau,r,t)\nabla p(\xi,\tau) - \nabla p^*(\xi,\tau,r,t)p(\xi,\tau)\big)d\Gamma\, d\tau.$$

When the represented point is moved to the boundary with a limiting process in space and time, and functions are discretized on the boundary [13], equation of motion in the matrix form is constructed as,

$$H_{nn}p_n - G_{nn}q_n = -\sum_{m=1}^{n-1} H_{nm}p_m + \sum_{m=1}^{n-1} G_{nm}q_m + f_n$$

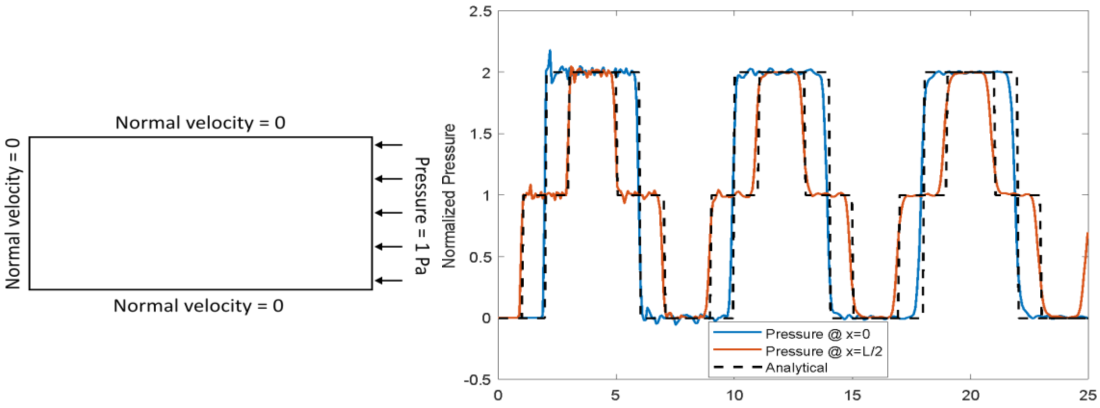
where the index $n$ refers to the $n^{th}$ discrete time step and the summation is coming due to the convolution nature of the representation formula. Solution for the unknowns at time step $n$ can be obtained by first obtaining the solution for the earlier time steps $m$ starting from the initial time. This time marching procedure can be quite expensive to calculate and store.

## 1.2 Implementation & Results

Due to lack of any time domain BEM open source code, implementation starts with linear elements with classical Lagrange first order shape functions used in FEM in 2 dimensions. Implemented method is verified through two examples with analytical solution.

First example is one dimensional discontinuous wave propagation through a rectangular body, by applying a sudden pressure from one end while the other ends are sound hard boundaries (i.e. zero Neuman boundary condition). Problem is defined in the Figure 1 (a) and the resulting time history of two points located at the left end and the middle can be seen in Figure 1 (b) compared with an analytical solution with 1D rod assumptions. Sudden jump in pressure is captured satisfactorily considering variables are linearly changing in time. Due to numerical damping introduced in the method, solution gets smoother at later times which ensures the stability of the result.

***Figure 1:*** *One dimensional rod example's domain (a) and the time response for two points inside the domain compared with analytical solution (b)*




(a) (b)

Second example is used to verify the code for the exterior (unbounded) domains, where a unit circle is excited through all its boundaries with a time harmonic signal and the response obtained at $R = 1.1, 2, 4$ units are compared with the analytical solution as given in Figure 2. Again solution is satisfactory except for a small amplitude error caused by discretization error of the boundary since IGA is not yet applied to the method.

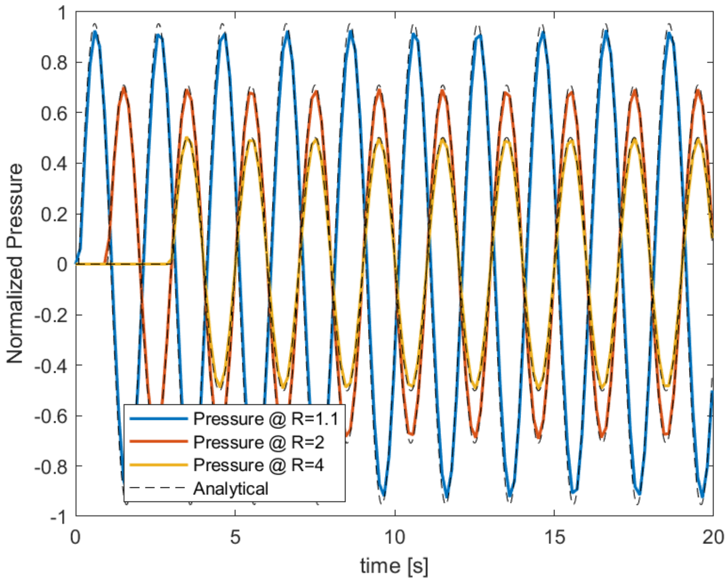



***Figure 2:*** *Two dimensional exterior sound radiation from pulsating circle's response on three points compared with analytical solution.*

## 2. Model Order Reduction

Most industrial scale problems require huge computational effort or storage unless the problem is very simple due to scaling problems. Hence, a reduction in size for the model might be needed especially for the problems with repetitive solutions, and one of the most common methods to achieve it is model order reduction (MOR) [14]. In MOR, system inputs and outputs are projected to a lower dimensional space that captures most related information within the aim of the solution, shown mathematically as,

$$K_r = W^T K V \quad with \quad x_r = V^T x \quad and \quad f_r = W f$$

where $x$, $f$, and $K$ are the outputs, inputs, and system matrix, respectively; while $W$ and $V$ are the projection matrices obtained with different methods with different goals in mind depending on the application or the requirement of the problem.

MOR is specially advantageous for time domain BEM since the storage of each time domain's system matrix is required which might get expensive even for relatively smaller sized problems. There are several advances to reduce the size of the problem in time direction in the literature such as interpolating the fundamental solution in time [15]. However, for the problems that already require MOR to store one or two system matrices this method should also be combined with other methods such as Automatic Krylov Recycling [16].

# 3. CONCLUSIONS

In this report transient acoustic analysis is researched and investigated for in BEM and FEM context. Furthermore time domain acoustic BEM implementation is verified with academic examples in 2D with linear elements. Advantages of applying IGA workflows is considered and highlighted however no result in that research direction is presented. Similarly, possible advantages and application direction for MOR is highlighted. In the future IGA and MOR methods will be applied on the already developed tools to obtain novel results.

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