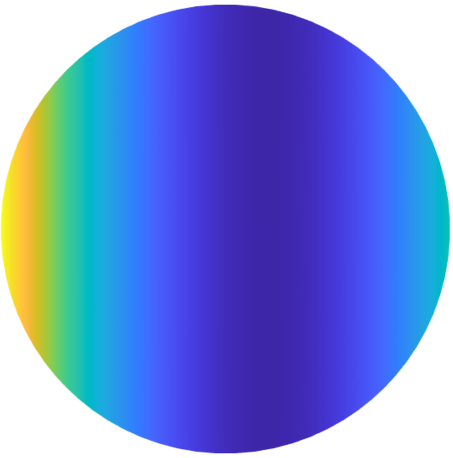
Gecko
Design for IGA-type discretization workflows

Gecko Technical Report 1

DC09 – Philip Le



This project has received funding from the European Union's Horizon Europe research and innovation programme under grant agreement No 101073106 Call: HORIZON-MSCA-2021-DN-01

Funded by the European Union

Executive summary

Numerical methods like the Boundary Element Method (BEM) are often employed predict the acoustic performance of components. However, since within such techniques the generation of a mesh is required, the actual geometry is replaced by a mere approximation, thus reducing the accuracy of predictions. To ensure geometric exactness, isogeometric analysis (IGA) which uses Non-Uniform Rational B-splines (NURBS) to represent the geometry, are used in combination with the BEM (IGABEM). Nevertheless, employing the BEM within the IGA framework (IGABEM) induces a high computational cost, due to the dense and frequency-dependent nature of the BEM systems. In that context, this report presents model order reduction strategies for the acoustic boundary element method within the IGA framework to speed-up the computational time and alleviate the memory costs.

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AKR	*Automatic Krylov Recycling*
BEM	*Boundary Element Method*
CAD	*Computer Aided Design*
IGA	*IsoGeometric Analysis*
FEM	*Finite Element Method*
MoR	Model order reduction
NVH	*Noise vibration harshness*
ROM	*Reduced order model*

Introduction

Due to increasing regulation regarding noise radiation and vibration, the acoustic performance of components have become a key factor in the product design cycle. Computer-aided design (CAD) is usually the first step in the virtual product design cycle to digitize the geometrical features of a product. For the prediction of the NVH characteristics of the product, traditional element-based methods are used and require the conversion of the CAD geometry into an analysis-suitable format. To obtain an analysis model, the CAD geometry is discretized by replacing its geometry with a piecewise polynomial approximation. Usually, low order polynomial approximations are used for this. This process is called meshing. Generating a suitable mesh for the considered geometry can be a complex process for design engineers and introduces some approximation errors and inaccuracies. Furthermore, any design change of the geometry requires another meshing process resulting into analysing a new model. For problems of industrial complexity, where the analysis steps are integrated in an optimization process, this process can drastically slow down the product development cycle. According to Hughes et al. [1] the procedure can take up to 80% of the overall analysis time.

To alleviate this shortcoming and ensure geometric exactness, Hughes et al. [1] introduced Isogeometric Analysis (IGA) in 2005 which aims to bridge the gap between CAD and computer aided engineering (CAE) by integrating the Finite-Element Analysis (FEA) and CAD within one workflow. Instead of utilizing low-order polynomial shape functions for the discretization process in the traditional FEA, IGA adopts the concept of using identical basis functions for computation and for CAD operations [2]. In recent years, extensive research has been conducted to apply IGA in various engineering domains such as structural mechanics [3], contact mechanics [4], computational fluid dynamics [5], FSI-problems [6] and acoustics [7,8,9]. For the implementation of IGA, different types of Spline functions like T-Splines, Non-Uniform Rational B-Splines (NURBS) have been used to represent the geometry in engineering designs.

In the research of computational acoustics, Simpson et al. [7] combined the direct collocational Boundary Element Method (BEM) approach with the IGA methodology which utilizes T-Splines to solve linear time-harmonic acoustic problems. In comparison to the Finite Element Method (FEM), employing the BEM is well suited for modelling unbounded acoustic problems. The BEM inherently satisfies the Sommerfeld radiation condition and deals with a smaller number of Degrees of Freedom (DoFs) due to the need of only discretizing the boundary surface instead of the volume. The direct collocational BEM within the IGA framework, however, is limited to solve closed boundary surfaces only. Therefore enabling the solution of industrial applications to solve combined exterior/interior acoustic problems, e.g. open boundary problems, in 2014 Coox et al. [9] developed an indirect variational formulation BEM in conjunction with IGA (IGABEM) based on NURBS.

For large-scale industrial applications where the analysed geometry is complex and a frequency analysis over a specific range is needed, numerical methods reach their computational limits. Especially the BEM induces a high computational cost in comparison to FEM due to the dense and frequency-dependent nature of its corresponding systems [10]. To alleviate this computational burden, Model Order Reduction (MOR) techniques have been utilized to find low-order models while maintaining a good approximation of the full-order model (FOM) over a desired frequency range. Proper orthogonal decomposition (POD) [11], the reduced basis method (RBM) [12] and Krylov moment matching [13] are common MOR techniques utilized in recent research. In the field of acoustics, MOR techniques have been employed in the context of vibro-acoustics using FEM. Van de Walle et al. [14] and Cai et al. [15], for instance, use Krylov moment matching which matches the first moments of the low order polynomials system in order to preserve stability in the time domain.

For the BEM, the application of standard MOR techniques in a straightforward manner as in FEM is hindered due to the frequency dependency of the BEM systems, leading to a tedious procedure to create a representative projection basis. In addition, every parameter value change, i.e. frequency change, requires the creation of a new projection basis, and hence diminishes the purpose of MOR. Recently, different techniques such as Krylov subspaces recycling have been employed to accelerate the acoustic analyses using BEM [16].

In this research, an automatic MOR scheme based on Krylov subspaces recycling [17] is applied in conjunction with IGABEM to solve acoustic problems. The automatic MOR method automates the selection of Krylov subspaces to be recycled and creates a projection basis which sufficiently approximates the solution of the FOM. The projection basis is used in combination with a Chebyshev polynomial approximation to create a reduced order model (ROM), thus alleviating the computational burden.

1 Model Order Reduction of acoustic isogeometric BEM systems

1.1 Acoustic Fundamentals

To describe the steady-state dynamic behaviour in the acoustic domain, a differential equation is needed. The governing differential equation for linear acoustic problems is the Helmholtz-equation which is used to compute the sound pressure $p(\boldsymbol{r})$ in the acoustic domain

$$\Delta\, p(\boldsymbol{r}) + k^2 p(\boldsymbol{r}) = -j\rho\omega q \delta(\boldsymbol{r}, \boldsymbol{r_q}), \qquad r \in \Omega$$

where $k = \omega/c$ is the acoustic wavenumber, ω is the angular frequency, c is the speed of sound, q is the strength of an acoustic volume velocity source at a position $\boldsymbol{r_q}$. The mathematical terms used above are as follow: $j^2 = -1$ is the imaginary unit, $\delta(i,j)$ denotes the Dirac-delta function, $\Delta = \nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denotes the Laplace operator.

The indirect variational boundary element method is based on the indirect boundary integral formulation which solves the pressure difference and normal pressure gradient between two sides of the boundary instead of directly solving for the acoustic pressure $p(\boldsymbol{r})$. The difference in pressure between two sides is called double layer potential $\mu(\boldsymbol{r_f})$ and the single layer potential $\sigma(\boldsymbol{r_f})$ is defined as the difference of the normal pressure gradient between two sides of the boundary Γ [21].

To compute the single and double layer potential, $\sigma(\boldsymbol{r_f})$ and $\mu(\boldsymbol{r_f})$ respectively, on the boundary surface, the boundary conditions are enforced using the indirect boundary integral formulation, leading to three integral equations. By applying a weighted residual formulation of these equations, a variational formulation can be obtained by

$$\forall\, (\delta\sigma, \delta\mu){:} \int_{\Gamma_p} R_p(\sigma,\mu)\delta\sigma d\Gamma + \int_{\Gamma_v} R_v(\sigma,\mu)\delta\mu d\Gamma + \int_{\Gamma_Z} R_Z(\sigma,\mu)\delta\mu d\Gamma = 0,$$

where $R_p(\sigma,\mu)$, $R_v(\sigma,\mu)$, $R_Z(\sigma,\mu)$ are the boundary residuals for the Dirichlet, Neumann and Robin boundaries, respectively. The expressions are omitted for the sake of brevity and can be found in []. By discretizing the variational formulation numerically, a symmetric system of equations can be obtained in the form of

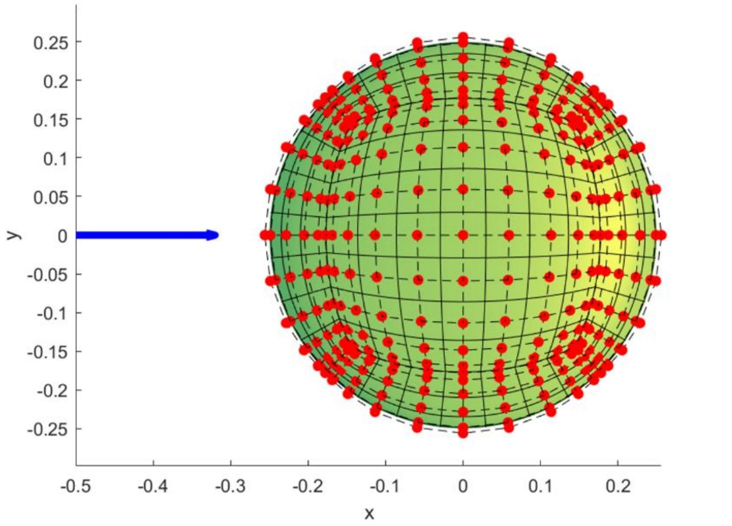
$$\boldsymbol{A}(\omega)\boldsymbol{x}(\omega) = \boldsymbol{b}(\omega), \qquad \omega \in \Psi,$$

where $\boldsymbol{A}: \Psi \to \mathbb{C}^{N\times N}$ is the symmetric system matrix and $\boldsymbol{b}, \boldsymbol{x}: \Psi \to \mathbb{C}^N$ are the force vector resulting from the imposed variables and the vector of unknown potentials, $\sigma(\boldsymbol{r_f})$ and $\mu(\boldsymbol{r_f})$, respectively.

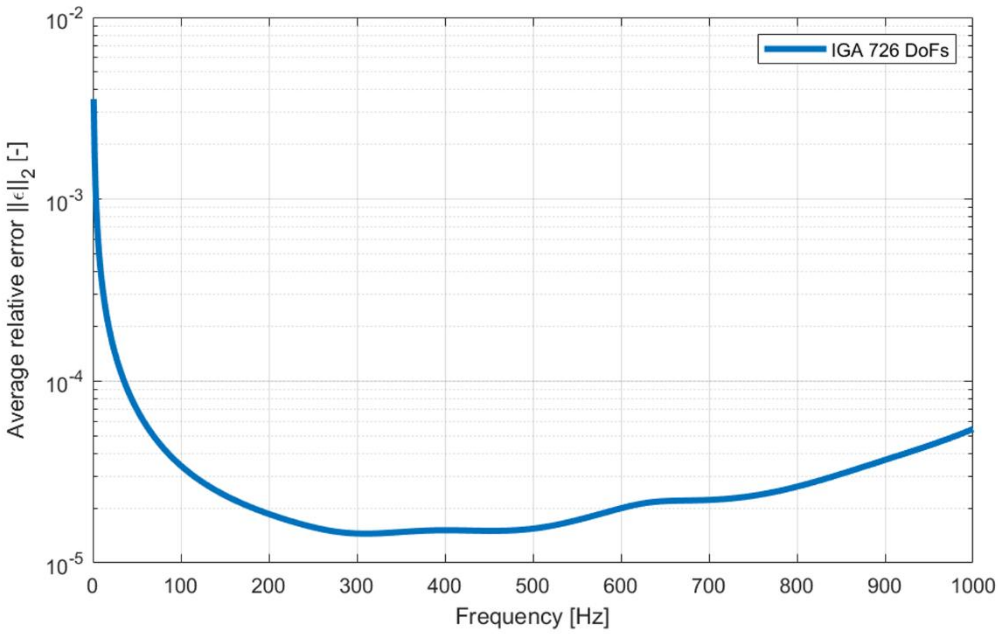
1.2 AKR MOR in IGA-BEM

The performance of the presented method is investigated and verified by an analytical solution in this section.

In Figure 1 (a) a sphere with a radius r = 0.25 m is illustrated. A plane wave with an amplitude of 1.0 is propagating through an unbounded air volume (ρ_0= 1.225 kg/m^3 , c = 340 m/s) and is impinging from the left on a rigid sphere ($\bar{v}_n$ = 0 m/s for the entire boundary surface). The acoustic pressure is studied in a frequency range from 1 to 1000 Hz with a step size of 1 Hz leading to a grid of values $\Phi \in \Psi$ with $|\Phi| = 1000$. The sphere consists of 6 identical conforming, biquartic patches and is modeled with NURBS polynomial of degree 4 which results to a total of 726 DoFs. The acoustic pressure $\boldsymbol{p}(\boldsymbol{r})$ is measured at 50 different positions ranging from r to 5r.



(a)



(b)

Figure 1: *(a) Plane wave scattering problem by a rigid sphere; (b) relative error of scattering pressure*

To validate the FOM from the numerical analysis using IGABEM for the plane wave scattering problem by a rigid sphere, the average relative error of the acoustic pressures with respect to the reference solution is investigated in Figure 1 (b). The reference solution refers to the analytical solution of the plane wave scattering problem by a rigid sphere which can be computed as in [19]. The L2 relative error norm on the radiated pressure field $\boldsymbol{p}(\boldsymbol{r})$ is computed.

Throughout the analysed frequency range the average relative error is below 10^{-2}, showing that IGABEM is accurate. For low frequencies up to 50 Hz a high increase of relative error is seen which can be associated to the system being numerically ill-conditioned.

For the reduced order model (ROM) the frequency sweep is performed by employing the AKR method in combination with the Chebyshev polynomial approximation. The projection basis is constructed in the offline phase. Following, the procedure of [19] the maximum distance of the sphere d_{max} := 0.5 m and a degree $\mathcal{M}$:= 25 is considered to provide sufficient accuracy. The normalized residual is used as an error estimator to check the accuracy of the ROM with regards to the FOM. The residual threshold is set at $r_{tol} := 10^{-2}$. The SPL from the FOM and ROM in Figure 2 show no differences, implying that the considered threshold r_{tol} is sufficient. By inspecting the normalized residual for the defined frequency range, the residual is below the predefined tolerance of 10^{-2}. To generate the projection basis V_{AKR}, the AKR algorithm only requires 2 partial solves and 3 full assemblies. The AKR algorithm produces a basis spanning a subspace of 62 dimensions leading to more drastic reductions compared to the FOM with a dimension of 726.

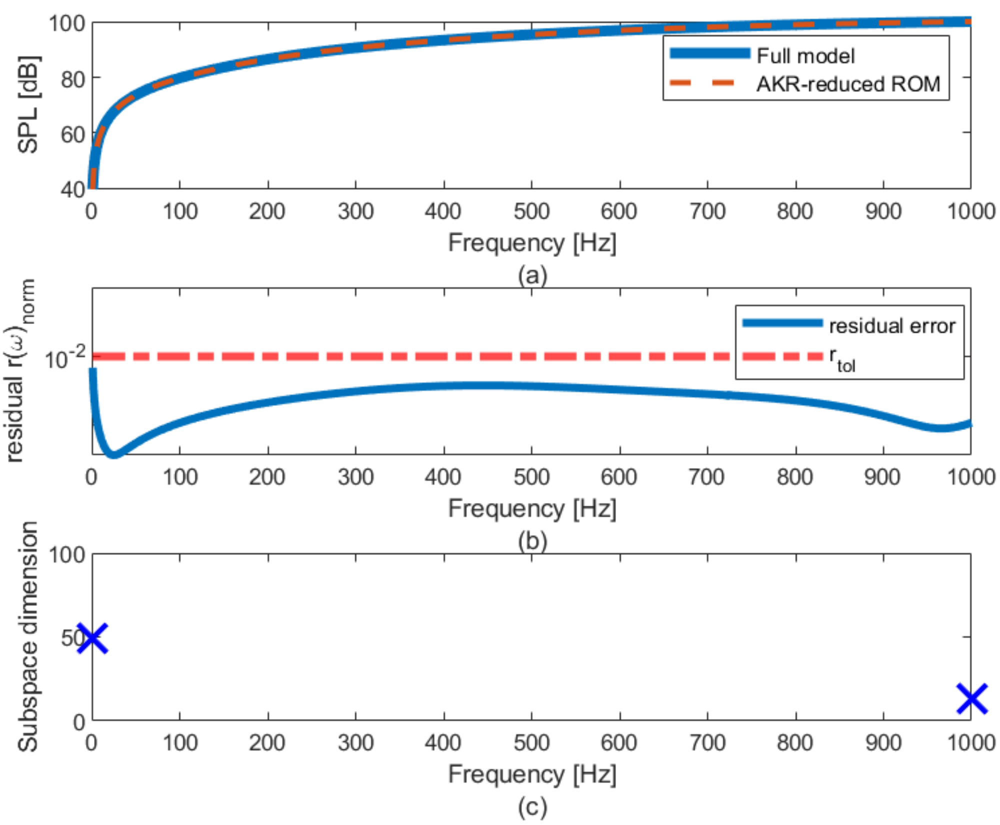


***Figure 2**: (a) Sound pressure level at an evaluated point; (b) Normalized residual error of the double layer potential (ROM); (c) Configuration for AKR with $x(\omega), \omega \in \Omega$*

Besides inspecting the accuracy of the ROM, it is also important to analyze the computation cost of the IGABEM, the Chebyshev polynomial approximation of the IGABEM system (Cheb-IGABEM) and the ROM induced by the MOR method.

Comparing the computational cost in the offline phase in Table 1, shows that both the ROM and the Cheb-IGABEM require similar computational cost, whereas the IGABEM has no offline cost. The greatest cost in the offline phase is related to the construction of the frequency-independent parameters $\boldsymbol{T}_i$ and $\boldsymbol{q}_i$.

Inspecting the total wall-clock time, shows that the largest speed-up for the Cheb-IGABEM and ROM is related to the assembly of the procedure. For both the ROM and Cheb-IGABEM the computational time is accelerated by avoiding the assembly procedure for every frequency. The ROM is slightly superior than the Cheb-IGABEM in terms of total cost due to the small size of the system and hence cheaper assembly procedure.

In terms of storage the ROM outperforms the Cheb-IGABEM and the IGABEM. For the ROM only the reduced frequency independent parameter $\boldsymbol{T}_{i,red}$ needs to be stored, whereas for Cheb-IGABEM the initial frequency independent parameter $\boldsymbol{T}_i$ has to be stored to solve for the interested frequency range. For large-scale problems the reduction of computational time and memory is more pronounced for the AKR-reduced ROM.

Operation	Cheb-IGABEM	IGABEM	ROM
Offline Cost			
Construction of projection basis	-	-	17 min
Construction of Chebyshev polynomial	80 min	-	62 min
Total wall-clock time	91 min	63h 36 min	81 min
Memory	150 MB	5.8 MB	2.1 MB

Table 1: *Computational cost for a frequency sweep of the plane wave scattering problem*

2 CONCLUSIONS

In this research, a model order reduction technique was applied for acoustic IGABEM using NURBS shape functions. The IGA-BEM is first verified by an analytical solution of the plane wave scattering problem by a rigid sphere. Afterwards, based on the FOM of the IGA-BEM, the presented technique uses Chebyshev polynomials to express the IGA-BEM system in an affine expression and a Galerkin projection is deployed for the order reduction of the resulting affine system. The reduction basis is created by leveraging the Krylov subspaces recycling of [18]. The combination of methods is analysed in terms of accuracy, computational cost and memory usage and comparisons with the baseline techniques are performed

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