Gecko
Design for IGA-type discretization workflows

Gecko Technical Report 2

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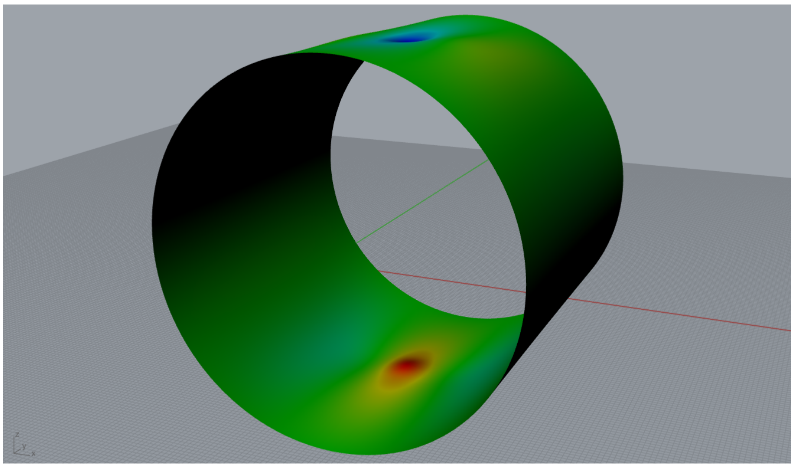

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Executive summary

This technical report presents the implementation of a Reissner Mindlin shell element within the Isogeometric B-Rep Analysis framework as part of the GECKO project, which aims to bridge the gap between Computer-Aided Design and Computer-Aided Engineering workflows. The work addresses the implementation of structural elements for large deformation analysis, with an initial focus on shell elements.

The first phase has delivered a functional Reissner Mindlin shell element for linear elastic materials, validated through standard benchmark cases, all integrated within the existing Kratos Multiphysics framework. While the core functionality is in place, challenges remain in complex geometries and the need for refined integration schemes. The next phase will therefore focus on locking mitigation, trimming and multipatch coupling features.

This work contributes to the GECKO project's broader goal of developing an integrated CAD-CAE workflow that maintains geometric accuracy while providing efficient and reliable analysis capabilities for industrial problems. This initial implementation lays the foundation for future simulation of thin-walled structures under large deformations, with the ultimate goal of improving the design and analysis process.

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List of abbreviations

CAD	*Computer-Aided Design*
CAE	*Computer-Aided Engineering*
FEA	*Finite Element Analysis*
IBRA	*Isogeometric Boundary Representation Analysis*
IGA	*Isogeometric Analysis*
KL	*Kirchhoff-Love*
RM	*Reissner-Mindlin*
PK	*Piola-Kirchhoff*
NURBS	*Non-Uniform Rational B-Splines*
MITC	*Mixed Interpolation of Tensorial Components*
ANS	*Assumed Natural Strain*
DSG	*Discrete Shear Gap*
B BAR	*Strain projection method*
6p	*Three-parameter (KL type, three displacements)*
3p	*Six-parameter (RM type, three displacements + three director rotations)*

Introduction

The seamless integration of CAD and CAE remains one of the central challenges in modern industrial workflows. Traditional finite element analysis requires geometry preprocessing and mesh generation, which creates an unnecessary gap between design and analysis. IBRA addresses this gap by using the same geometric description for both design and analysis, thereby preserving geometric exactness and reducing preprocessing effort.

This work contributes to the development umbrella of robust structural elements for large deformation analysis within the IBRA framework. Shell elements are an appropriate starting point, since thin-walled structures are central in many industrial applications and represent an important step toward direct CAD integrated simulation workflows. The implementation is carried out in Kratos Multiphysics.

This deliverable outlines a systematic approach, starting with some background, moving to the methodology, validation of the implementation highlighting the current work, future directions, and conclusion of the report.

1. Background

Thin-walled shells are central to many of the industrial structures targeted by GECKO, and their midsurface representation aligns naturally with the surface based NURBS models exchanged with CAD in IBRA. The present work begins from KL shell element existing infrastructure and extends it to a RM shell formulation for 3D linear isotropic materials on flat and curved geometries. On flat patches, the director can be treated in a simpler manner, but curved geometries require more adaptations. Refinement is performed through standard IGA operations, such as knot insertion and degree elevation, which increase the control point resolution without changing the exact geometry.

When the formulation was extended, a central numerical challenge became apparent: locking. Although NURBS bases can reduce locking compared with low order finite elements, they do not remove it completely. This makes its treatment a necessary next development step rather than a refinement detail. In parallel multipatch coupling, since many industrial models cannot be represented robustly with a single untrimmed patch. These extensions introduce additional challenges due to interface coupling, local parameterization, and integration on trimmed domains all interact with locking and boundary condition enforcement.

2. Methodology

2.1 Element formulation

The implementation follows a degenerated solid RM formulation with 6p per control point: three translations of the midsurface and three rotations describing the shell director orientation. In contrast to KL shells, the director is treated as an independent kinematic field, which relaxes the normality constraint and allows transverse shear deformation. This makes the formulation suitable for moderately thick shells and for later extensions to nonlinear constitutive behaviour.

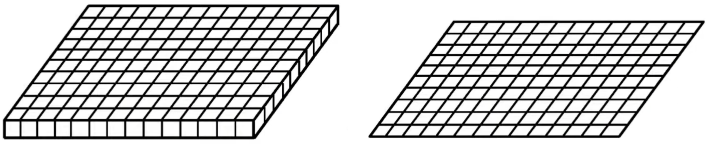

Figure 1 Mid surface assumption in shells

The shell continuum is parameterised as

$$\mathbf{X}(\xi^1, \xi^2, \zeta) = \mathbf{x}(\xi^1, \xi^2) + \zeta\, \mathbf{d}(\xi^1, \xi^2),$$

with x the midsurface position and $\mathbf{d}$ the unit director. The covariant base vectors $\mathbf{a}_\alpha = \partial_\alpha \mathbf{x}$ and the director $\mathbf{a}_3 = \mathbf{d}$ define the local frame; the contravariant base $\mathbf{a}^\alpha$ and the metric tensors $a_{\alpha\beta}, a^{\alpha\beta}$ follow in the standard way. The second fundamental form $b_{\alpha\beta} = -\mathbf{a}_\alpha \cdot \partial_\beta \mathbf{a}_3$ carries the curvature contribution. The director $\mathbf{d}$ introduced is treated as an independent kinematic field, so the rotation of the cross section is incorporated directly into the displacement formulation rather than recovered from displacement derivatives as in the KL case.The element stores covariant metric terms, curvature terms, and the local base vectors at each Gauss point, and the surface normal is computed from the cross product of the covariant tangent vectors rather than assumed a priori. Geometry and discretisation are described by NURBS basis functions, providing the higher order continuity that distinguishes IGA from classical C^0 finite elements [13]. Strains are obtained from the Green Lagrange tensor expressed in curvilinear coordinates and split into membrane part, bending part, and transverse shear part that KL neglects

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathrm{m}} + \zeta\, \boldsymbol{\varepsilon}^{\mathrm{b}} + \boldsymbol{\gamma}^{\mathrm{s}},$$

The constitutive response is the St. Venant-Kirchhoff model under a plane-stress assumption through the thickness (consistent with linear elastic, small deformation conditions) giving the 2nd PK stress $\mathbf{S} = \mathbb{C}\, \boldsymbol{\varepsilon}$ with $\mathbb{C}$ the plane-stress elasticity tensor built from Young's modulus E and Poisson's ratio ν. Internal forces are obtained by through thickness integration of $\mathbf{S}$ over the shell volume. Equilibrium is enforced in the standard weak form: find the displacement field such that the virtual work of internal and external forces balances for all admissible variations of x and $\mathbf{d}$. Following [2], the rotational DOFs of $\mathbf{d}$ are parameterized through two independent angles, which keeps the unknowns at five per control point and avoids the need to update a full rotation tensor.

The element uses a 6p RM kinematics and assembles membrane, bending, and drilling contributions in the stiffness matrix. The drilling term is included through a stabilization coefficient that suppresses spurious in plane rotational modes while preserving the displacement based shell formulation. This makes the element numerically more robust without changing the underlying shell kinematics.

2.2 Locking phenomena

Pure displacement based shell elements suffer from shear and membrane locking in the thin-shell limit, with stiffness scaling that overwhelms the bending response as the slenderness ratio grows. The pathology and a unified taxonomy are treated in detail in Koschnick's dissertation [9]. In IGA the higher polynomial degrees and increased continuity of NURBS basis functions partly alleviate locking, but they do not remove it.

A range of treatments has been proposed in the IGA shell literature, each with different tradeoffs. Hierarchic formulations [5] separate KL, RM, and 3D shell kinematics, eliminating transverse shear locking by construction. Projection based methods [6][7] replace locking sensitive strain components by their projection onto a reduced strain space. MITC [11] and ANS [10] formulations reconstruct shear strains from values sampled at tying points. Reduced and Greville-type quadrature rules [12] lower the number of integration points while preserving accuracy in spline discretization. DSG-type strategies [8] are conceptually attractive but harder to adapt efficiently to multipatch NURBS settings.

These strategies differ not only in the numerical mechanism used to alleviate locking, but also in their implementation burden within the existing Kratos shell infrastructure. Projection-based and reduced quadrature approaches are comparatively simple to integrate because they preserve much of the existing displacement-based structure. In contrast, MITC/ANS and mixed formulations require additional element-level data handling, modified interpolation or auxiliary fields, and more careful treatment of the strain-displacement operators. DSG-type methods are particularly attractive conceptually, but they are harder to transfer robustly to multipatch NURBS settings, where interface coupling, local parameterization, and integration consistency become nontrivial. For this reason, the evaluation of these methods is guided by both formulation complexity and the effort required to adapt them cleanly within Kratos.

Table 1 Locking-treatment strategies for IGA shells

Strategy	Principle	Formulation / Kratos complexity
Projection based methods	Replace locking sensitive strain components by their projection onto a reduced strain space	Low element local and minimally intrusive
MITC / ANS	Reconstruct shear and, where needed, membrane strains from values sampled at tying points.	Moderate requires a degree dependent tying point scheme
Mixed Hellinger-Reissner	Introduce independent strain or stress fields that are condensed locally at the element level.	Moderate auxiliary fields increase formulation and implementation complexity, but the displacement interface remains standard
Reduced / Greville-type quadrature	Use spline specific quadrature rules with fewer integration points while maintaining accuracy	Low minimal formulation changes, but quadrature handling must remain consistent with the existing Kratos integration pipeline
DSG-type	Reconstruct strains from discrete shear gaps	High limited IGA track record and more challenging adaptation to the Kratos shell infrastructure

2.3 Implementation details

The implementation was developed in Kratos Multiphysics by extending the existing shell infrastructure. The initial version was restricted to flat geometries and linear isotropic materials to validate the kinematics and constitutive operators. After successful patch-test verification, the formulation was extended to curved geometries. The integration weights were also modified to account for curvature-dependent Jacobians. Refinement is performed through knot insertion and degree elevation, which increase the control-point resolution without changing the geometry.

The element is implemented in the IGA application of Kratos Multiphysics. The 6p kinematics follow the degenerated solid construction of [2], reusing the geometric infrastructure of the existing three parameter shell element [3] for basis evaluation, knot span queries, and quadrature. Curvilinear to local Cartesian transformations of the strain tensor and constitutive matrix are performed at each Gauss point, pre-and post-processing of geometry use the Rhino/Cocodrilo toolchain.

In parallel, locking mitigation strategies are being developed for both the existing 3 parameters KL shell and the six parameters RM shell. The work is organized around several contributions, including a mixed HR variational reformulation, a B-bar strain projection, and the geometry enforcement issues that arise for single patch shells with strong boundary conditions via the penalty terms, as well as DSG approaches that are noted as a complementary avenue.

3. Validation

The cantilever patch test provides an initial verification of the shell kinematics under both in plane and out of plane loading. The comparison between the KL and RM versions shows nearly identical displacements in the shell regime, which confirms that the new element is consistent with the existing shell infrastructure in this limit.

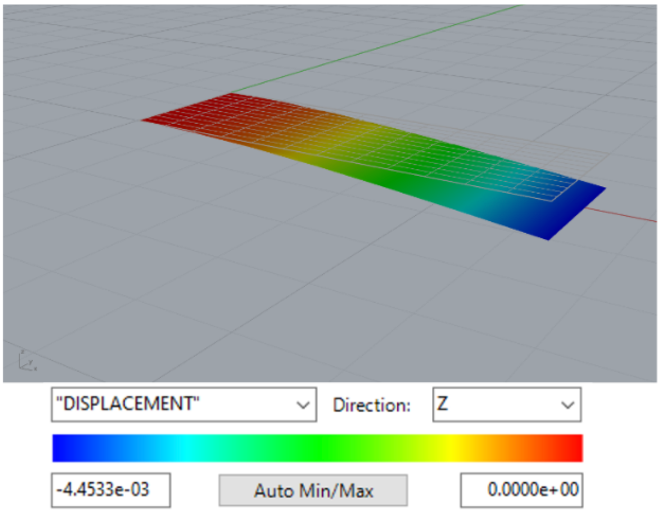

Figure 2. Cantilever with Kirchoff Love elements

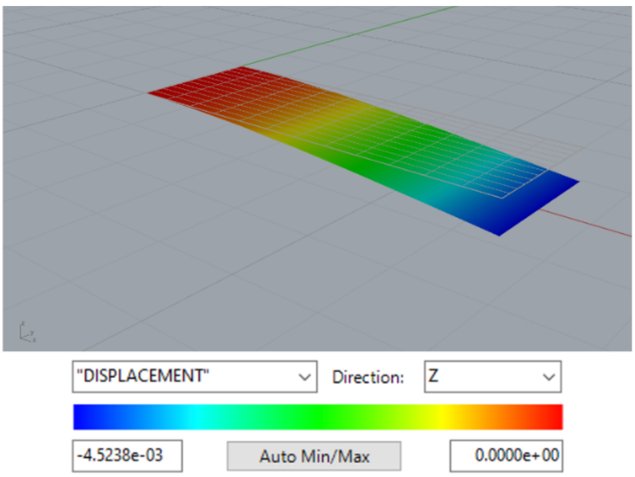

Figure 3. Cantilever with Reissner Mindlin elements

The Scordelis Lo roof is a standard benchmark for curved shell behaviour and for the interaction between membrane and bending effects. The computed mid-span displacement compares well with the reference value (displacement at the mid-span point u=0.3006) reported in the literature, and the convergence trend confirms that the curved geometry formulation is functioning correctly. Small fluctuations at certain polynomial degrees suggest that the formulation is numerically stable but still sensitive to the choice of discretization and refinement.

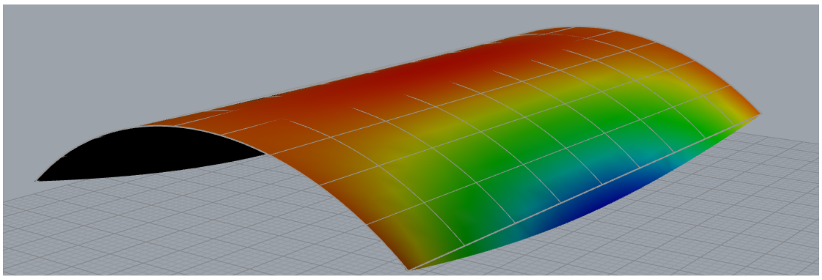

Figure 4. Scordelis Lo Roof, full model with Reissner Mindlin shell elements

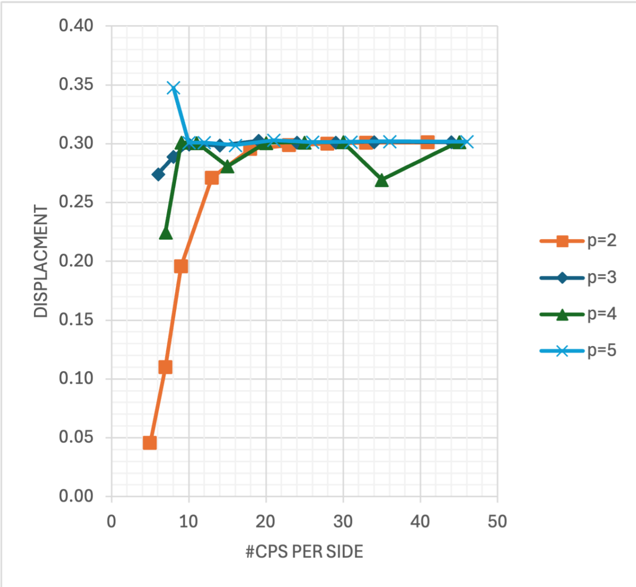

Figure 5. Convergence of the displacement for the Scordelis Lo Roof, full model

Table 2 Benchmark parameters

Case	Geometry	Material	Loading
Scordelis Lo roof	$R = 25.0$, $L = 50.0$, $t = 0.25$, half angle $40°$	$E = 4.32 \times 10^8$, $\nu = 0$	Self weight, $g = 90$

Following some recent updates to the element formulation, the case was re-run on a quarter model that exploits the two planes of symmetry of the geometry and loading.

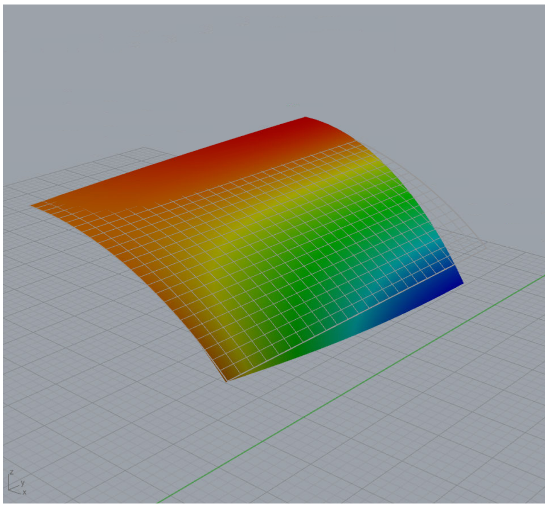

Figure 6 Scordelis Lo roof, quarter model with Reissner Mindlin shell elements

The computed midspan displacement converges cleanly to the reference value. Higher polynomial degrees recover the reference within a handful of control points per side, while $p = 2$ still requires noticeably more refinement before converging. The fluctuations observed earlier are no longer present, and all four polynomial degrees collapse onto the reference by roughly ten control points per side.

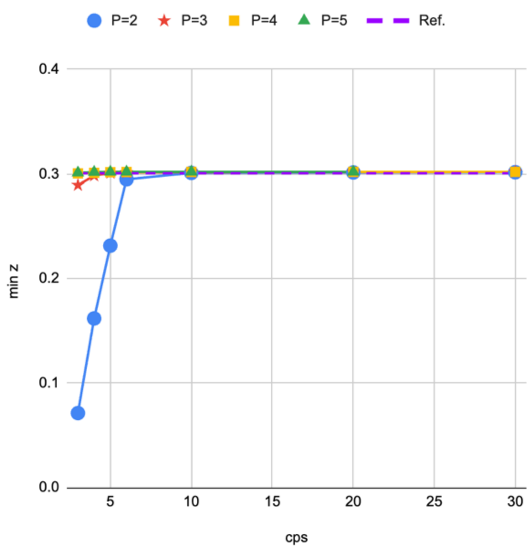

Figure 7 Convergence of the displacement for the Scordelis Lo Roof quarter model

With the formulation now validated on the untrimmed quarter model, the next step is to exercise the same benchmark with multipatch coupling and trimming.

The pinched cylinder is more demanding than the Scordelis Lo roof because it strongly exposes membrane bending coupling and is the classical benchmark for locking sensitive thin-shell behavior. Following the formulation update, the case was also re-run on a reduced computational domain, exploiting the symmetry of the geometry and loading

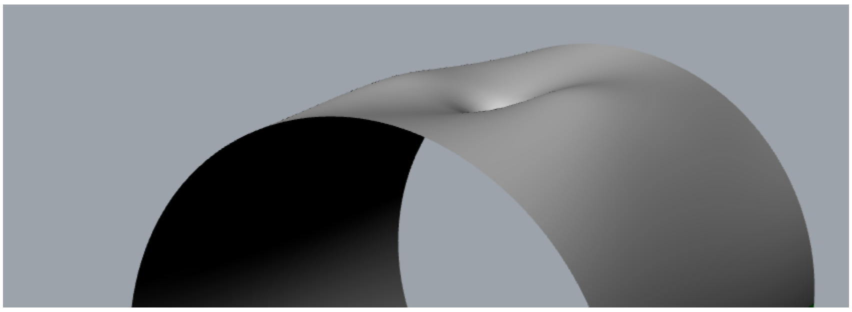

Figure 8. Pinched Cylinder, full model with Reissner Mindlin shell elements

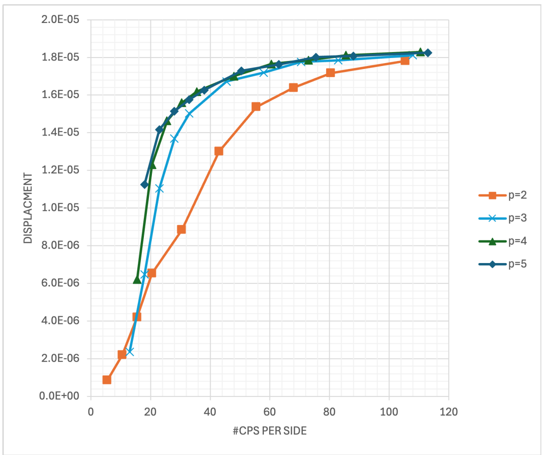

Figure 9. Convergence of the displacement for the pinched cylinder full model

Table 3 Benchmark parameters

Case	Geometry	Material	Loading
Pinched cylinder	$R = 300.0$, $L = 600.0$, $t = 3.0$	$E = 3.0 \times 10^6$, $\nu = 0.3$	Two diametrically opposite point loads, $P = 1.0$

The implemented element captures the correct global deformation pattern at all polynomial degrees, and the load point displacement converges toward the reference value $u_{\text{ref}} = 1.8264 \times 10^{-5}$. Although we see that it needs more refinement to do so.

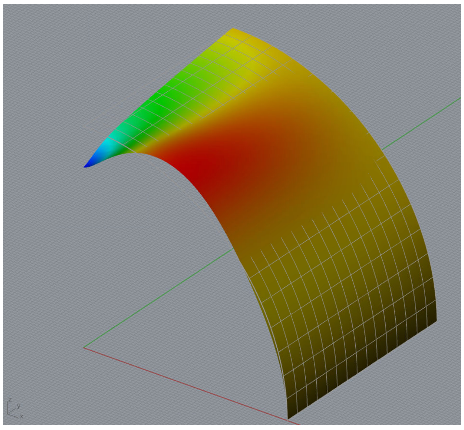

Figure 10 Pinched Cylinder, quarter model with Reissner Mindlin shell elements

In the case of quarter, the convergence behavior converges to the reference value much quicker than previously, however, it is strongly degree dependent, $p \geq 4$ reaches the reference within a handful of control points per side, $p = 3$ requires roughly twice as much refinement, and $p = 2$ converges much more slowly and only approaches the reference at the finest meshes tested. This is the expected signature of a possibility of locking in a displacement based RM formulation; higher order NURBS bases smooth the discrete strain space and alleviate the locking modes, but they do not eliminate them, and low degree discretizations remain visibly affected.

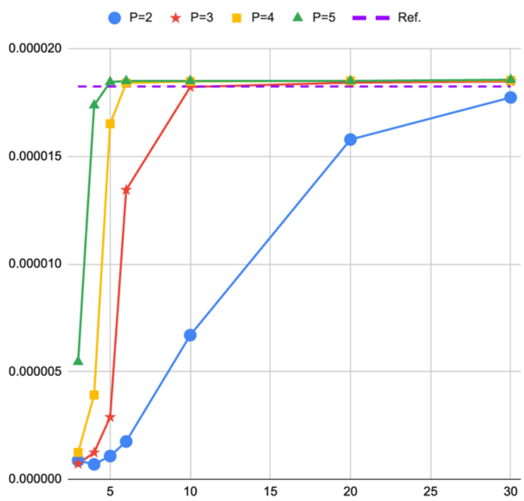

Figure 11 Convergence of the displacement for the pinched cylinder quarter model

A dedicated locking treatment will be introduced in the next phase. The effect is also expected to interact with the weak coupling and trimmed integration domains foreseen for the multipatch and trimmed extensions, which makes a treatment that does not rely on simply increasing the polynomial degree all the more important.

4. Current limitations and future work

The current implementation provides a functional 6p Reissner Mindlin shell element for linear elastic analysis of flat and curved geometries. However, several limitations remain before the formulation can be considered robust for broader use. First, the element is still sensitive to refinement in locking prone cases, which motivates the introduction of dedicated locking treatments. Second, trimming and multipatch coupling are still in progress and must be completed to support realistic CAD derived geometries. Third, large deformation effects are not yet fully included and will be addressed in the longer term through a geometrically nonlinear kinematic formulation. Fourth, the present material model is restricted to linear elasticity, so damage and plasticity will also need to be introduced at a later stage.

The next development stages will therefore proceed in a staged manner. The immediate priority is to stabilize the shell formulation on locking benchmarks and evaluate candidate locking treatments in terms of formulation complexity, implementation effort, and compatibility with the existing Kratos shell infrastructure. Multipatch coupling and trimming are the next major milestones, since they are essential for more general geometry handling.

5. Conclusion

This work has established a functional Reissner Mindlin shell element within the Kratos IGA framework and validated it on standard benchmark cases. The formulation performs well for flat and curved geometries, but the results also show that locking, boundary condition mis enforcement, and refinement sensitivity remain important issues in demanding shell problems.

The current implementation should therefore be viewed as a baseline rather than a final formulation. Its main value is that it makes the remaining challenges visible in a controlled setting and provides a platform for the next development steps, which will focus on locking mitigation, multipatch coupling, and large deformation kinematics. These extensions are necessary to move from a validated prototype toward a formulation that is suitable for industrial geometries.

Overall, the work supports the broader GECKO goal of narrowing the CAD-CAE gap through accurate and geometry consistent structural analysis.

6. References

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