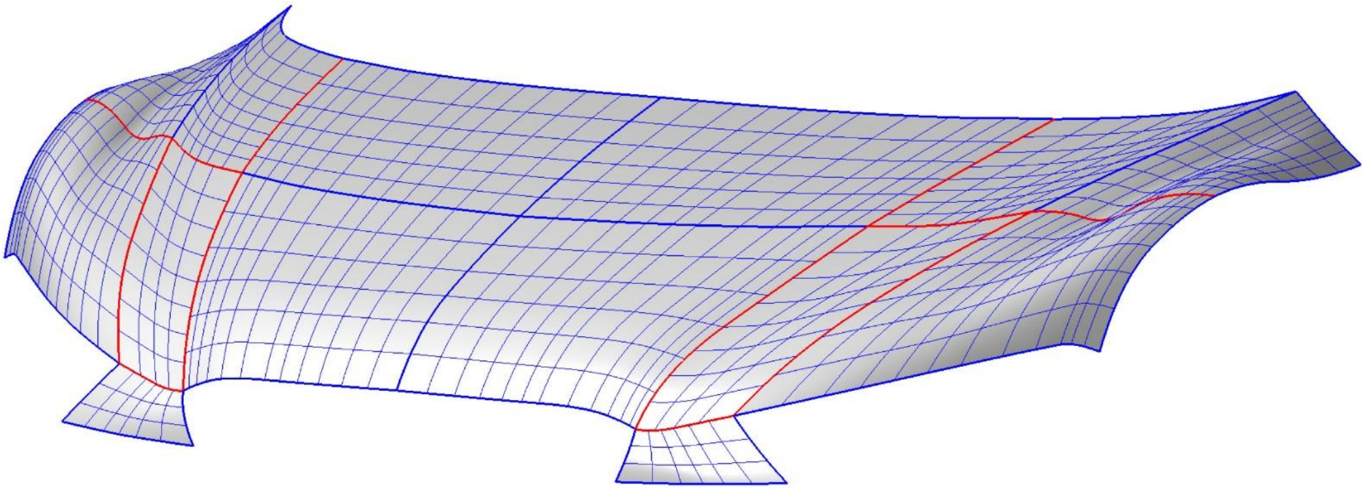
Gecko
Design for *IGA*-type
discretization workflows

Gecko Technical Report 1

DC09 – Philip Le



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Executive summary

Numerical methods like the Boundary Element Method (BEM) are often employed predict the acoustic performance of components. However, since within such techniques the generation of a mesh is required, the actual geometry is replaced by a mere approximation, thus reducing the accuracy of predictions. To ensure geometric exactness, isogeometric analysis (IGA) which uses Non-Uniform Rational B-splines (NURBS) to represent the geometry, are used in combination with the BEM (IGABEM). Nevertheless, employing the BEM within the IGA framework (IGABEM) induces a high computational cost, due to the dense and frequency-dependent nature of the BEM systems. In that context, this report presents model order reduction strategies for the acoustic boundary element method within the IGA framework to speed-up the computational time and alleviate the memory costs.

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List of abbreviations

AKR	*Automatic Krylov Recycling*
BEM	*Boundary Element Method*
CAD	*Computer Aided Design*
IGA	*IsoGeometric Analysis*
FEM	*Finite Element Method*
MoR	Model order reduction
NVH	*Noise vibration harshness*
ROM	*Reduced order model*

Introduction

Due to increasing regulation regarding noise radiation and vibration, the acoustic performance of components have become a key factor in the product design cycle. Computer-aided design (CAD) is usually the first step in the virtual product design cycle to digitize the geometrical features of a product. For the prediction of the NVH characteristics of the product, traditional element-based methods are used and require the conversion of the CAD geometry into an analysis-suitable format. To obtain an analysis model, the CAD geometry is discretized by replacing its geometry with a piecewise polynomial approximation. Usually, low order polynomial approximations are used for this. This process is called meshing. Generating a suitable mesh for the considered geometry can be a complex process for design engineers and introduces some approximation errors and inaccuracies. Furthermore, any design change of the geometry requires another meshing process resulting into analysing a new model. For problems of industrial complexity, where the analysis steps are integrated in an optimization process, this process can drastically slow down the product development cycle. According to Hughes et al. [1] the procedure can take up to 80% of the overall analysis time.

To alleviate this shortcoming and ensure geometric exactness, Hughes et al. [1] introduced Isogeometric Analysis (IGA) in 2005 which aims to bridge the gap between CAD and computer aided engineering (CAE) by integrating the Finite-Element Analysis (FEA) and CAD within one workflow. Instead of utilizing low-order polynomial shape functions for the discretization process in the traditional FEA, IGA adopts the concept of using identical basis functions for computation and for CAD operations [2]. In recent years, extensive research has been conducted to apply IGA in various engineering domains such as structural mechanics [3], contact mechanics [4], computational fluid dynamics [5], FSI-problems [6] and acoustics [7,8,9]. For the implementation of IGA, different types of Spline functions like T-Splines, Non-Uniform Rational B-Splines (NURBS) have been used to represent the geometry in engineering designs.

In the research of computational acoustics, Simpson et al. [7] combined the direct collocational Boundary Element Method (BEM) approach with the IGA methodology which utilizes T-Splines to solve linear time-harmonic acoustic problems. In comparison to the Finite Element Method (FEM), employing the BEM is well suited for modelling unbounded acoustic problems. The BEM inherently satisfies the Sommerfeld radiation condition and deals with a smaller number of Degrees of Freedom (DoFs) due to the need of only discretizing the boundary surface instead of the volume. The direct collocational BEM within the IGA framework, however, is limited to solve closed boundary surfaces only. Therefore enabling the solution of industrial applications to solve combined exterior/interior acoustic problems, e.g. open boundary problems, in 2014 Coox et al. [9] developed an indirect variational formulation BEM in conjunction with IGA (IGABEM) based on NURBS.

For large-scale industrial applications where the analysed geometry is complex and a frequency analysis over a specific range is needed, numerical methods reach their computational limits. Especially the BEM induces a high computational cost in comparison to FEM due to the dense and frequency-dependent nature of its corresponding systems [10]. To alleviate this computational burden, Model Order Reduction (MOR) techniques have been utilized to find low-order models while maintaining a good approximation of the full-order model (FOM) over a desired frequency range. Proper orthogonal decomposition (POD) [11], the reduced basis method (RBM) [12] and Krylov moment matching [13] are common MOR techniques utilized in recent research. In the field of acoustics, MOR techniques have been employed in the context of vibro-acoustics using FEM. Van de Walle et al. [14] and Cai et al. [15], for instance, use Krylov moment matching which matches the first moments of the low order polynomials system in order to preserve stability in the time domain.

For the BEM, the application of standard MOR techniques in a straightforward manner as in FEM is hindered due to the frequency dependency of the BEM systems, leading to a tedious procedure to create a representative projection basis. In addition, every parameter value change, i.e. frequency change, requires the creation of a new projection basis, and hence diminishes the purpose of MOR. Recently, different techniques such as Krylov subspaces recycling have been employed to accelerate the acoustic analyses using BEM [16].

In this research, an efficient two-step MOR scheme based on Krylov subspaces recycling [17] is applied in conjunction with IGABEM to solve acoustic problems. The automatic MOR method automates the selection of Krylov subspaces to be recycled and creates a projection basis which sufficiently approximates the solution of the FOM. The projection basis is used in combination with a Chebyshev polynomial approximation to create a reduced order model (ROM), thus alleviating the computational burden. A second reduction step is employed using the reduced basis method to further compress the ROM size and allow a fast on-line phase.

1 Model Order Reduction of acoustic isogeometric BEM systems

1.1 Acoustic Fundamentals

To describe the steady-state dynamic behaviour in the acoustic domain, a differential equation is needed. The governing differential equation for linear acoustic problems is the Helmholtz-equation which is used to compute the sound pressure $p(\boldsymbol{r})$ in the acoustic domain

$$\Delta\, p(\boldsymbol{r}) + k^2 p(\boldsymbol{r}) = -j\rho\omega q\delta(\boldsymbol{r}, \boldsymbol{r_q}), \qquad r \in \Omega$$

where $k = \omega/c$ is the acoustic wavenumber, ω is the angular frequency, c is the speed of sound, q is the strength of an acoustic volume velocity source at a position $\boldsymbol{r_q}$. The mathematical terms used above are as follow: $j^2 = -1$ is the imaginary unit, $\delta(i, j)$ denotes the Dirac-delta function, $\Delta = \nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denotes the Laplace operator.

The indirect variational boundary element method is based on the indirect boundary integral formulation which solves the pressure difference and normal pressure gradient between two sides of the boundary instead of directly solving for the acoustic pressure $p(\boldsymbol{r})$. The difference in pressure between two sides is called double layer potential $\mu(\boldsymbol{r_f})$ and the single layer potential $\sigma(\boldsymbol{r_f})$ is defined as the difference of the normal pressure gradient between two sides of the boundary Γ [21].

To compute the single and double layer potential, $\sigma(\boldsymbol{r_f})$ and $\mu(\boldsymbol{r_f})$ respectively, on the boundary surface, the boundary conditions are enforced using the indirect boundary integral formulation, leading to three integral equations. By applying a weighted residual formulation of these equations, a variational formulation can be obtained by

$$\forall\ (\delta\sigma, \delta\mu) \colon \int_{\Gamma_p} R_p(\sigma, \mu)\delta\sigma d\Gamma + \int_{\Gamma_v} R_v(\sigma, \mu)\delta\mu d\Gamma + \int_{\Gamma_Z} R_Z(\sigma, \mu)\delta\mu d\Gamma = 0,$$

where $R_p(\sigma, \mu)$, $R_v(\sigma, \mu)$, $R_Z(\sigma, \mu)$ are the boundary residuals for the Dirichlet, Neumann and Robin boundaries, respectively. The expressions are omitted for the sake of brevity and can be found in [18]. By discretizing the variational formulation numerically, a symmetric system of equations can be obtained in the form of

$$\boldsymbol{A}(\omega)\boldsymbol{x}(\omega) = \boldsymbol{b}(\omega), \qquad \omega \in \Psi,$$

where $\boldsymbol{A} \colon \Psi \to \mathbb{C}^{N \times N}$ is the symmetric system matrix and $\boldsymbol{b}, \boldsymbol{x} \colon \Psi \to \mathbb{C}^{N}$ are the force vector resulting from the imposed variables and the vector of unknown potentials, $\sigma(\boldsymbol{r_f})$ and $\mu(\boldsymbol{r_f})$, respectively.

1.2 Affine approximation of the IGABEM system

In this work, Chebyshev polynomials are employed to approximate the IGABEM system as well as providing a uniform error distribution in Ψ [10]. By leveraging a Chebyshev polynomial approximation for each $\boldsymbol{A}(\omega)$ and $\boldsymbol{b}(\omega)$, the system can be displayed as

$$\left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) T_i\right) x(\omega) = \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega) q_i,$$

where $c_i(\omega)$, with $i = 0, \dots \mathcal{M}$, represent the Chebyshev polynomials of the first kind. The prime indicates that the first term is halved. By taking account the maximum dimension of the model and the desired frequency range, the truncation order of the Chebyshev polynomial approximation can be predetermined as suggested in [19]. The frequency independent parameters of the polynomial $\boldsymbol{T}_i \in \mathbb{C}^{N\times N}$ and are defined as $\boldsymbol{q}_i \in \mathbb{C}^{N}$ follows

$$T_i = \frac{2}{\mathcal{M}+1}\sum_{i=0}^{\mathcal{M}} \boldsymbol{A}(\omega_k)c_i(\omega_k),$$

$$q_i = \frac{2}{\mathcal{M}+1}\sum_{i=0}^{\mathcal{M}} \boldsymbol{b}(\omega_k)c_i(\omega_k),$$

where the positions of the Chebyshev nodes ω_k correspond to the n zeros of the polynomials in the interval $[-1\ 1]$and are obtained by

$$\omega_k = \cos\left(\frac{\pi\left(k+\frac{1}{2}\right)}{\mathcal{M}+1}\right).$$

To compute the true position of the Chebyshev nodes for $\omega \in \Psi$, a linear mapping of $\Psi \rightarrow (-1,1)$ is needed. The assembly procedure can be expressed by a linear combination of the coefficient matrices with the Chebyshev polynomials of first kind, thus accelerating the assembly procedure. However, by transforming the non-affine IGABEM system in an affine way, higher memory storage is required as all the dense frequency independent parameters $\boldsymbol{T}_i$ and $\boldsymbol{q}_i$ must be stored.

1.3 Galerkin model order reduction

After approximating the original BEM system in an affine manner, a Galerkin projection can be exploited to reduce the size of the system matrix and thus tackle the high storage cost. In the Galerkin MOR, the quantity of interest $\mathbf{x}(\omega)$ is approximated by a solution $\hat{\mathbf{x}} \in \mathcal{W}$ with $\mathcal{W} \coloneqq \operatorname{span}\{\mathbf{W}\}$ which then can be expressed by a linear combination of linearly independent column vectors of $\mathbf{W} \in \mathbb{C}^{N\times l}$

$$\mathbf{x}(\omega) \approx \hat{\mathbf{x}}(\omega) = \mathbf{W}\tilde{\mathbf{x}}(\omega),$$

where $\tilde{\mathbf{x}}(\omega)$ is the reduced solution vector Imposing the Galerkin condition $\mathbf{r}(\omega) \coloneqq \mathbf{b}(\omega) - \mathbf{A}(\omega)\hat{\mathbf{x}}(\omega) \perp \mathbf{W}$, a one-sided or Galerkin projection approach is constituted to regain a square form. By employing this strategy, the following one-sided projection can be obtained

$$\mathbf{W}^H\mathbf{A}(\omega)\mathbf{W}\tilde{\mathbf{x}}(\omega) = \mathbf{W}^H\mathbf{b}(\omega),$$

where $\mathbf{W}^H$ is the Hermitian of $\mathbf{W}$. Combining the Galerkin projection in conjunction with the Chebyshev polynomial approximation of the IGABEM system, the equation can be reformulated as

$$\left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega)\mathbf{W}^H\mathbf{T}_i\mathbf{W}\right)\tilde{\mathbf{x}}(\omega) = \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega)\mathbf{W}^H\mathbf{q}_i,$$

yielding a compact form of

$$\left(\sum_{i=0}^{\mathcal{M}}{}' c_i(\omega)\boldsymbol{T}_{i,redAKR}\right)\tilde{\boldsymbol{x}}(\omega) = \sum_{i=0}^{\mathcal{M}}{}' c_i(\omega)\boldsymbol{q}_{iredAKR},$$

where $\boldsymbol{T}_{i,redAKR} \in C^{l \times l}$ and $\boldsymbol{q}_{iredAKR} \in C^{l}$ are the frequency-independent coefficients of the reduced polynomial system and $l \coloneqq rank(W)$. Projecting each frequency-independent parameter $\boldsymbol{T}_i$ and $\boldsymbol{q}_i$ right after its assembly, reduces the size of the parameters and consequently, ensures that the memory requirements remain within the limits of the IGABEM.

1.4 Automatic Krylov subspace recycling

In order to preserve the high fidelity property of the FOM, this work utilizes the AKR algorithm to create an initial global reduction basis $\mathbf{W}$ with $\mathcal{W} = span\{\mathbf{W}\}$. The AKR algorithm is based on Krylov subspaces recycling and offers a fast construction of a high-quality basis $\mathbf{W}$, by keeping the number of system assemblies and full solutions in the off-line phase of the MOR method to a minimum. Instead of assembling and solving $\forall\, \omega \in \Phi$, where Φ is a discrete set with $|\Phi| = N_{\text{tot}}$ and N_{tot} is the number of systems to be resolved, the AKR algorithm recycles Krylov subspaces only for $\omega \in \Omega_M$, where $\Omega_M \coloneqq [\omega_1, \dots, \omega_L]$ is a discrete ordered set with $\Omega_M \subset \Psi$ and $|\Omega_M| \ll |\Phi|$. The residual $r(\omega)$ is used as an error estimator and evaluated during the construction of the basis $\boldsymbol{W}$ to guarantee a high fidelity. For a given frequency range $\Psi \coloneqq [\omega_{\min}, \omega_{\max}]$ as well as a residual threshold r_{thresh}, the residual $\boldsymbol{r}_i(\omega)$ is computed as

$$\mathbf{r}_{\mathrm{i}}(\omega) = \mathbf{r}_0(\omega) - \boldsymbol{A} \quad (\omega)\tilde{\mathbf{x}}(\omega) \perp \mathbf{W}$$

where $\mathbf{r}_0(\omega)$ denotes the initial residual for $\mathbf{x}_0(\omega)$. The residual $\mathbf{r}_{\mathrm{i}}(\omega)$ is computed for each iteration i and is checked to satisfy $\left|\left|\mathbf{r}(\omega)\right|\right| \leq r_{\text{thresh}} \quad \forall\, \omega \in \Psi$, where

$$\tilde{\mathbf{x}}_{\text{AKR}}(\omega) \in \mathbf{x}_0(\omega) + \mathcal{W}$$

with $\boldsymbol{x}_0 \colon \Psi \to \mathrm{C}^{\mathrm{N}}$an initial guess of the solution. Without loss of generality, $\boldsymbol{x}_0 := 0$ is assumed. The Krylov subspaces are generated through an Arnoldi algorithm [20] and represented as

$$\mathcal{K}_{m(\omega_j)}(\mathbf{A}(\omega_j), \mathbf{r}_0(\omega_j)) \coloneqq \; span\,\{\boldsymbol{r}_0(\omega_j), \mathbf{A}(\omega_j)\mathbf{r}_0(\omega_j), \dots,$$

$$\mathbf{A}^{m-2}(\omega_j)\mathbf{r}_0(\omega_j), \mathbf{A}^{m-1}(\omega_j)\mathbf{r}_0(\omega_j)\} \; = \; span\{\mathbf{V}_{\omega_j}\},$$

where $m(\omega_j)$ with $j \; = \; 1, \dots, L$ denotes the dimension of the respective Krylov subspaces for each $\omega_i \in \Omega_M$ and $L \; \coloneqq \; |\Omega_M|$. A modified Gram-Schmidt orthogonalizaton is performed to prevent numerical instabilities [21]. A global reduction basis W spans the Krylov subspaces

$$span\{\mathbf{W}\} = \mathcal{K}^{\omega_1}_{m(\omega_1)} \cup \mathcal{K}^{\omega_2}_{m(\omega_2)} \cup \dots \cup \mathcal{K}^{\omega_L}_{m(\omega_L)}.$$

The dimension $m(\omega_j)$ of the Krylov subspaces $\mathcal{K}_{m(\omega_j)}$ and Ω_M are determined iteratively through the AKR algorithm.

1.5 Efficient reduction scheme

Utilizing the single-step reduction with AKR-MOR results in only moderate reduction, as the number of Krylov subspaces to be recycled is dictated by the system's spectral properties, i.e. distribution of the eigenvalues, and the condition number. For highly resonant problems or ill-conditioned systems, the AKR-MOR does not offer substantial computational relief and the storage of the affine coefficient matrices becomes a bottleneck for the method [22].

To address this limiting factor and by taking advantage of the given reduced affine IGABEM system, a second reduction step utilizing RBM is employed to reduce the computational burden in memory and time further, enabling an efficient reduction. The RBM samples the reduced affine approximated IGABEM system and selects, following a greedy approach, the snapshots at frequency locations $\omega_{\bar{\kappa}+1} \in \Omega_{\mathrm{RBM}} \subset \Psi$, where Ω_{RBM} is a discrete ordered set with $|\Omega_{\mathrm{RBM}}| \ll |\Phi|$, to build a reduction basis. A residual based error estimator for a grid $\Phi \subset \Psi$ is used for the selection of snapshots [23].The residual error estimator is defined as

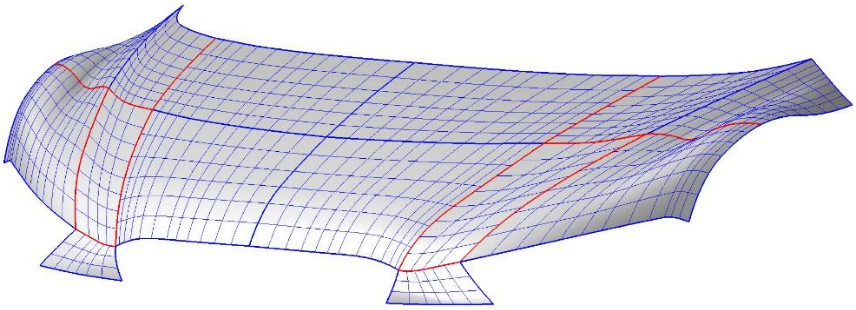
$$\mathbf{r}_{\mathrm{RBM},\bar{\kappa}}(\omega) := \mathbf{A}_{red}(\omega)\overline{\boldsymbol{V}}_{\bar{\kappa}}\tilde{\mathbf{x}}_{\mathrm{2Step}}(\omega) - \mathbf{b}_{\mathrm{red}}(\omega), \forall\ \omega \in \Phi \subset \Psi,$$

where $\overline{\boldsymbol{V}}_{\bar{\kappa}}$ denotes the reduced basis from concatenating the $\bar{\kappa}$ snapshots. The residual $r_{\mathrm{RBM},\bar{\kappa}}$ is computed iteratively for all $\omega \in \Phi$. In each iteration, only the snapshot $\boldsymbol{x}_{\bar{\kappa}+1}$ at the location $\omega_{\bar{\kappa}+1}$ is chosen, which yields the worst approximation. The algorithm stops the selection until the computed residual $\mathbf{r}_{\mathrm{RBM},\bar{\kappa}}(\omega)$ is below a predefined threshold r_{tol}, i.e. $r_{\mathrm{RBM}} := \left\|\mathbf{r}_{\mathrm{RBM},\bar{\kappa}}(\omega)\right\| \leq r_{tol}, \forall\ \omega \in \Phi$ and returns the reduction basis $\boldsymbol{V}_{RBM} := \overline{\boldsymbol{V}}_{\bar{\kappa}}$ as output. During the on-line phase, the reduction basis V_{RBM} is used on top of the reduced affine expressed IGABEM system, enabling a fast computation and relieving the memory constraints further.

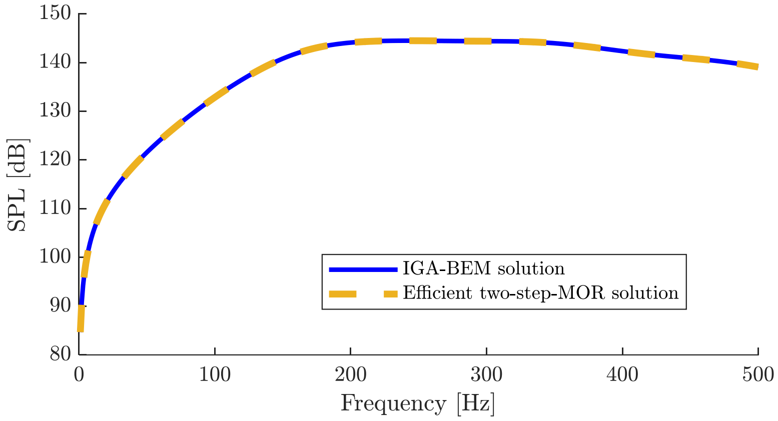
1.6 Results

The performance of the proposed two-step reduction method for acoustic IGABEM systems is investigated in terms of accuracy of the resulting ROMs and efficiency of the procedure of constructing the reduction bases. A car hood structure is studied. The dimension of the model are roughly $1.8 \times 1.4 \times 0.4\ m$ and it consists of 14 NURBS patches with NURBS of polynomial degree 2. The structure of the car hood vibrates with an unit velocity ($\bar{v}_n = 1m/s$ across the entire surface) and the sound propagates through the unbounded acoustic domain. The acoustic pressure is measured at a microphone which is located at a distance of 0.6 m in height. The response is studied in a frequency range $\Psi := [1{,}500]$ Hz with a step size of 1 Hz, resulting in the sequential assembly and solution of 500 fully populated linear systems. In Figure 1(a) the non-conforming car hood is displayed and contains 1053 independent DOFs. The SPL response of the ROM produced by the two-step MOR is compared to the response by the FOM in Figure 1 (b), showing no differences, implying that the considered thresholds $r_{\mathrm{thresh}} := 10^{-2}$ and $r_{\mathrm{tol}} := 10^{-3}$ are sufficient. For the radiating car hood problem, a truncation order of $\mathcal{M} := 20$ was selected. By inspecting the normalized residuals of the two-step MOR method with respect to the FOM, it can be observed that the normalized residual stays below the residual threshold over the entire frequency band. The normalized residual produced by the approximated solution are displayed in Figure 2 (a). The corresponding sampling pattern for the construction $\Omega_M, \mathbf{W}$ and $\mathbf{V}_{\mathrm{RBM}}$ are displayed in Figure 2(b). The AKR algorithm requires only 7 full system assemblies and 4 partial solutions for the construction of $\mathbf{W}$. In addition, 20 full system assemblies are employed to construct the Chebyshev coefficient matrices. The resulting ROMs has a dimension of 13.

Besides inspecting the accuracy of the ROM, it is also important to analyze the computation cost of the IGABEM, the Chebyshev polynomial approximation of the IGABEM system (Cheb-IGABEM) and the ROM induced by the MOR method. As shown in Table 1, the highest computational cost for the MOR method is related to the construction of the affine coefficient matrices T_i and $\mathbf{q}_i$ which requires the full system assembly at the Chebyshev nodes. The resulting speed-up factors for the on-line stage demonstrate that the proposed method significantly accelerates both system assembly and solution compared to the FOM over the entire frequency range.

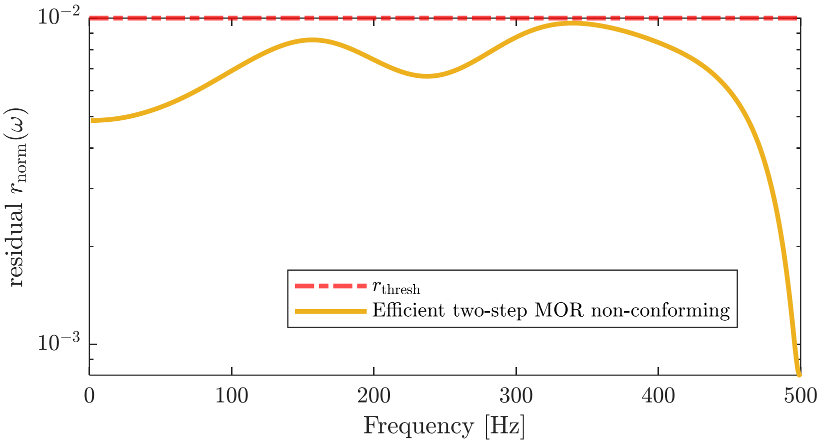


(a)

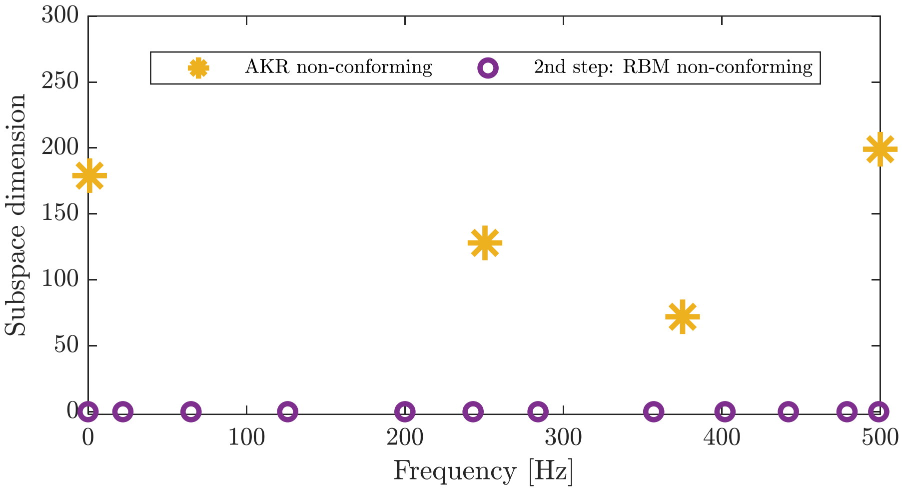


(b)

Figure 1: *(a) Car hood consisting of 14 NURBS patches of which some are non-conforming (indicated with red boundaries) [8]; (b) Response at (0, 0.7, 1)*



(a)



(b)

***Figure 1**: (a) Normalized residuals employing the proposed MOR method for the car hood problem; (b) Sampling pattern for the construction of the reduction base* $\mathbf{W}$ *and* $\mathbf{V}_{\mathrm{RBM}}$

Operation	IGABEM	ROM
Construction of projection basis W	-	36 min
Construction of Chebyshev polynomial	-	2h 20 min
Total assembly time	35h 50 min	24 s
Total solution time	25 s	0.3 s

***Table 1**: Computational cost for a frequency sweep for the car hood problem*

2 CONCLUSIONS

In this research, an efficient two-step reduction method was proposed for acoustic IGA indirect Galerkin BEM systems. The IGA-BEM is first verified by an analytical solution of the plane wave scattering problem by a rigid sphere. Afterwards, based on the FOM of the IGA-BEM, the presented technique uses Chebyshev polynomials to express the IGA-BEM system in an affine expression and a Galerkin projection is deployed for the order reduction of the resulting affine system. The reduction basis is created by leveraging the Krylov subspaces recycling of [18] and by using RBM as a second reduction step to compress the size further. The MOR method is analyzed in terms of accuracy and computational cost, showing that a substantial speed up in the on-line phase is offered.

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