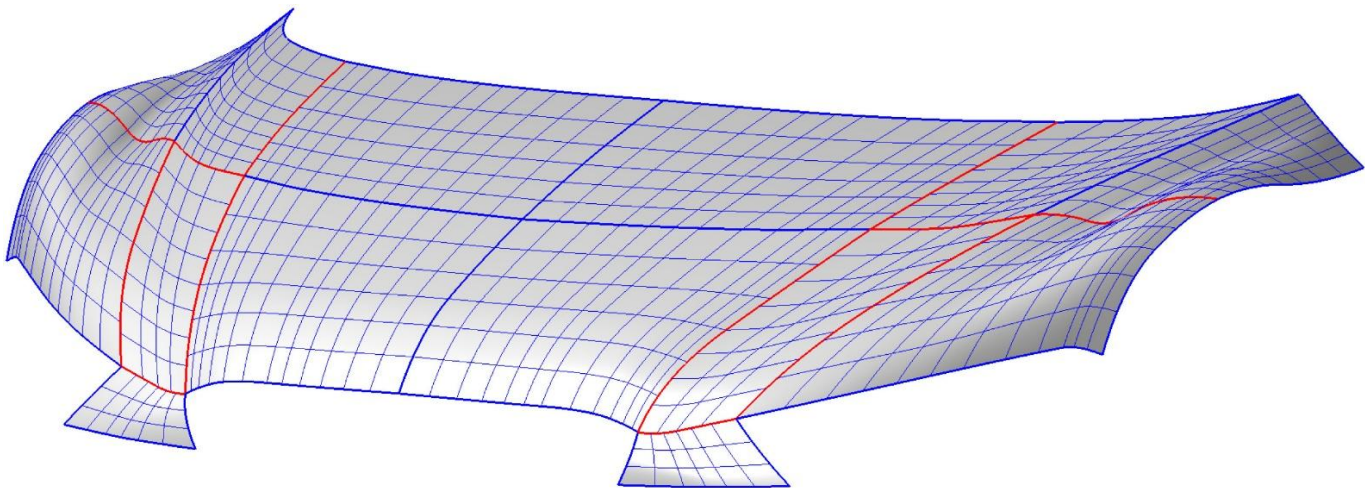




Gecko Technical Report 3

DC09 – Philip Le



This project has received funding from the European Union's Horizon Europe research and innovation programme under grant agreement No 101073106 Call: HORIZON-MSCA-2021-DN-01

Funded by the
European Union

Executive summary

Numerical methods like the Boundary Element Method (BEM) are often employed predict the acoustic performance of components. However, since within such techniques the generation of a mesh is required, the actual geometry is replaced by a mere approximation, thus reducing the accuracy of predictions. To ensure geometric exactness, isogeometric analysis (IGA) which uses Non-Uniform Rational B-splines (NURBS) to represent the geometry, are used in combination with the BEM (IGABEM). Nevertheless, employing the BEM within the IGA framework (IGABEM) induces a high computational cost, due to the dense and frequency-dependent nature of the BEM systems. In that context, this report presents model order reduction strategies for the acoustic boundary element method within the IGA framework to speed-up the computational time and alleviate the memory costs.

Table of contents

Executive summary	2
Table of contents	3
List of abbreviations	6
Introduction	7
1 Model Order Reduction of acoustic isogeometric BEM systems	9
1.1 Acoustic Fundamentals	9
1.2 Affine approximation of the IGABEM system	9
1.3 Galerkin model order reduction	10
1.4 Automatic Krylov subspace recycling	11
1.5 Efficient reduction scheme	11
1.6 Results	12
1.7 Burton-Miller formulation of the IGA indirect BEM and its application to MOR	14
1.8 Model order reduction for IGABEM	16
1.9 Particle swarm optimization for acoustic broadband frequencies using ROMs	17
2 CONCLUSIONS	20
3 REFERENCES	21

List of figures

Figure 1: (a) Car hood consisting of 14 NURBS patches of which some are non-conforming (indicated with red boundaries) [8]; (b) Response at $(0, 0.7, 1)$	13
Figure 2: (a) Normalized residuals employing the proposed MOR method for the car hood problem; (b) Sampling pattern for the construction of the reduction base $\mathbf{W}$ and $\mathbf{V}_{\text{RBM}}$	14
Figure 3: (a) Response at $(0.2750, 0, 0)$ between analytical and Burton-Miller IGA indirect Galerkin BEM, (b) Condition number of standard IGA indirect Galerkin BEM and Burton-Miller IGA indirect Galerkin BEM	13
Figure 4: (a) Normalized residual between the FOM using Burton-Miller IGABEM and its ROM, (b) Condition number of standard IGA indirect Galerkin BEM and Burton-Miller IGA indirect Galerkin BEM	16
Figure 5: Initial car hood shape with microphone arrays (marked as blue dots)	18
Figure 6: Patch indices with design variable location (marked as red crosses) for shape optimization	18
Figure 7: Convergence history of the particle swarm optimization MOR framework for broadband acoustic shape optimization of the car hood	18
Figure 8: Optimized shape of the car hood	19
Figure 9: SPL spectrum at microphone located $(0.6, -0.06, 0.7)$	19

List of tables

Table 1: Computational cost for a frequency sweep for the car hood problem	14
Table 2: Summary of the optimization setup and outcome for the non-conforming car hood problem	19

List of abbreviations

<i>AKR</i>	<i>Automatic Krylov Recycling</i>
<i>BEM</i>	<i>Boundary Element Method</i>
<i>CAD</i>	<i>Computer Aided Design</i>
<i>IGA</i>	<i>IsoGeometric Analysis</i>
<i>FEM</i>	<i>Finite Element Method</i>
<i>MoR</i>	Model order reduction
<i>NVH</i>	<i>Noise vibration harshness</i>
<i>ROM</i>	<i>Reduced order model</i>

Introduction

Due to increasing regulation regarding noise radiation and vibration, the acoustic performance of components has become a key factor in the product design cycle. Computer-aided design (CAD) is usually the first step in the virtual product design cycle to digitize the geometrical features of a product. For the prediction of the NVH characteristics of the product, traditional element-based methods are used and require the conversion of the CAD geometry into an analysis-suitable format. To obtain an analysis model, the CAD geometry is discretized by replacing its geometry with a piecewise polynomial approximation. Usually, low order polynomial approximations are used for this. This process is called meshing. Generating a suitable mesh for the considered geometry can be a complex process for design engineers and introduces some approximation errors and inaccuracies. Furthermore, any design change of the geometry requires another meshing process resulting into analysing a new model. For problems of industrial complexity, where the analysis steps are integrated in an optimization process, this process can drastically slow down the product development cycle. According to Hughes et al. [1] the procedure can take up to 80% of the overall analysis time.

To alleviate this shortcoming and ensure geometric exactness, Hughes et al. [1] introduced Isogeometric Analysis (IGA) in 2005 which aims to bridge the gap between CAD and computer aided engineering (CAE) by integrating the Finite-Element Analysis (FEA) and CAD within one workflow. Instead of utilizing low-order polynomial shape functions for the discretization process in the traditional FEA, IGA adopts the concept of using identical basis functions for computation and for CAD operations [2]. In recent years, extensive research has been conducted to apply IGA in various engineering domains such as structural mechanics [3], contact mechanics [4], computational fluid dynamics [5], FSI-problems [6] and acoustics [7,8,9]. For the implementation of IGA, different types of Spline functions like T-Splines, Non-Uniform Rational B-Splines (NURBS) have been used to represent the geometry in engineering designs.

In the research of computational acoustics, Simpson et al. [7] combined the direct collocational Boundary Element Method (BEM) approach with the IGA methodology which utilizes T-Splines to solve linear time-harmonic acoustic problems. In comparison to the Finite Element Method (FEM), employing the BEM is well suited for modelling unbounded acoustic problems. The BEM inherently satisfies the Sommerfeld radiation condition and deals with a smaller number of Degrees of Freedom (DoFs) due to the need of only discretizing the boundary surface instead of the volume. The direct collocational BEM within the IGA framework, however, is limited to solve closed boundary surfaces only. Therefore, enabling the solution of industrial applications to solve combined exterior/interior acoustic problems, e.g. open boundary problems, in 2014 Coox et al. [9] developed an indirect variational formulation BEM in conjunction with IGA (IGABEM) based on NURBS.

For large-scale industrial applications where the analysed geometry is complex and a frequency analysis over a specific range is needed, numerical methods reach their computational limits. Especially the BEM induces a high computational cost in comparison to FEM due to the dense and frequency-dependent nature of its corresponding systems [10]. To alleviate this computational burden, Model Order Reduction (MOR) techniques have been utilized to find low-order models while maintaining a good approximation of the full-order model (FOM) over a desired frequency range. Proper orthogonal decomposition (POD) [11], the reduced basis method (RBM) [12] and Krylov moment matching [13] are common MOR techniques utilized in recent research. In the field of acoustics, MOR techniques have been employed in the context of vibro-acoustics using FEM. Van de Walle et al. [14] and Cai et al. [15], for instance, use Krylov moment matching which matches the first moments of the low order polynomials system in order to preserve stability in the time domain.

For the BEM, the application of standard MOR techniques in a straightforward manner as in FEM is hindered due to the frequency dependency of the BEM systems, leading to a tedious procedure to create a representative projection basis. In addition, every parameter value change, i.e. frequency change, requires the creation of a new projection basis, and hence diminishes the purpose of MOR. Recently, different techniques such as Krylov subspaces recycling have been employed to accelerate the acoustic analyses using BEM [16].

In this research, an efficient two-step MOR scheme based on Krylov subspaces recycling [17] is applied in conjunction with IGABEM to solve acoustic problems. The automatic MOR method automates the selection of Krylov subspaces to be recycled and creates a projection basis which sufficiently approximates the solution of the FOM. The projection basis is used in combination with a Chebyshev polynomial approximation to create a reduced order model (ROM), thus alleviating the computational burden. A second reduction step is employed using the reduced basis method to further compress the ROM size and allow a fast on-line phase. Furthermore, the Burton-Miller formulation is incorporated into the MOR approach to address the non-uniqueness problem associated with exterior acoustic problems using the boundary integral formulation. Finally, the resulting MOR approach is applied to broadband acoustic shape optimization using a gradient free optimizer, demonstrating its applicability to frequency-sweep analyses during the optimization process.

1 Model Order Reduction of acoustic isogeometric BEM systems

1.1 Acoustic Fundamentals

To describe the steady-state dynamic behaviour in the acoustic domain, a differential equation is needed. The governing differential equation for linear acoustic problems is the Helmholtz-equation which is used to compute the sound pressure $p(\mathbf{r})$ in the acoustic domain

$$\Delta p(\mathbf{r}) + k^2 p(\mathbf{r}) = -j\rho\omega q\delta(\mathbf{r}, \mathbf{r}_q), \quad \mathbf{r} \in \Omega$$

where $k = \omega/c$ is the acoustic wavenumber, ω is the angular frequency, c is the speed of sound, q is the strength of an acoustic volume velocity source at a position $\mathbf{r}_q$. The mathematical terms used above are as follow: $j^2 = -1$ is the imaginary unit, $\delta(i, j)$ denotes the Dirac-delta function, $\Delta = \nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denotes the Laplace operator.

The indirect variational boundary element method is based on the indirect boundary integral formulation which solves the pressure difference and normal pressure gradient between two sides of the boundary instead of directly solving for the acoustic pressure $p(\mathbf{r})$. The difference in pressure between two sides is called double layer potential $\mu(\mathbf{r}_f)$ and the single layer potential $\sigma(\mathbf{r}_f)$ is defined as the difference of the normal pressure gradient between two sides of the boundary Γ [21].

To compute the single- and double-layer potential, $\sigma(\mathbf{r}_f)$ and $\mu(\mathbf{r}_f)$ respectively, on the boundary surface, the boundary conditions are enforced using the indirect boundary integral formulation, leading to three integral equations. By applying a weighted residual formulation of these equations, a variational formulation can be obtained by

$$\forall (\delta\sigma, \delta\mu): \int_{\Gamma_p} R_p(\sigma, \mu) \delta\sigma d\Gamma + \int_{\Gamma_v} R_v(\sigma, \mu) \delta\mu d\Gamma + \int_{\Gamma_z} R_z(\sigma, \mu) \delta\mu d\Gamma = 0,$$

where $R_p(\sigma, \mu)$, $R_v(\sigma, \mu)$, $R_z(\sigma, \mu)$ are the boundary residuals for the Dirichlet, Neumann and Robin boundaries, respectively. The expressions are omitted for the sake of brevity and can be found in [18]. By discretizing the variational formulation numerically, a symmetric system of equations can be obtained in the form of

$$\mathbf{A}(\omega)\mathbf{x}(\omega) = \mathbf{b}(\omega), \quad \omega \in \Psi,$$

where $\mathbf{A}: \Psi \rightarrow \mathbb{C}^{N \times N}$ is the symmetric system matrix and $\mathbf{b}, \mathbf{x}: \Psi \rightarrow \mathbb{C}^N$ are the force vector resulting from the imposed variables and the vector of unknown potentials, $\sigma(\mathbf{r}_f)$ and $\mu(\mathbf{r}_f)$, respectively.

1.2 Affine approximation of the IGABEM system

In this work, Chebyshev polynomials are employed to approximate the IGABEM system as well as providing a uniform error distribution in Ψ [10]. By leveraging a Chebyshev polynomial approximation for each $\mathbf{A}(\omega)$ and $\mathbf{b}(\omega)$, the system can be displayed as

$$\left(\sum_{i=0}^{\mathcal{M}} {}'c_i(\omega) T_i \right) x(\omega) = \sum_{i=0}^{\mathcal{M}} {}'c_i(\omega) q_i,$$

where $c_i(\omega)$, with $i = 0, \dots, \mathcal{M}$, represent the Chebyshev polynomials of the first kind. The prime indicates that the first term is halved. By taking account the maximum dimension of the model and the desired frequency range, the truncation order of the Chebyshev polynomial approximation can be predetermined as suggested in [19]. The frequency independent parameters of the polynomial $\mathbf{T}_i \in \mathbb{C}^{N \times N}$ and are defined as $\mathbf{q}_i \in \mathbb{C}^N$ follows

$$T_i = \frac{2}{\mathcal{M} + 1} \sum_{k=0}^{\mathcal{M}} \mathbf{A}(\omega_k) c_i(\omega_k),$$

$$q_i = \frac{2}{\mathcal{M} + 1} \sum_{k=0}^{\mathcal{M}} \mathbf{b}(\omega_k) c_i(\omega_k),$$

where the positions of the Chebyshev nodes ω_k correspond to the n zeros of the polynomials in the interval $[-1, 1]$ and are obtained by

$$\omega_k = \cos \left(\frac{\pi \left(k + \frac{1}{2} \right)}{\mathcal{M} + 1} \right).$$

To compute the true position of the Chebyshev nodes for $\omega \in \Psi$, a linear mapping of $\Psi \rightarrow (-1, 1)$ is needed. The assembly procedure can be expressed by a linear combination of the coefficient matrices with the Chebyshev polynomials of first kind, thus accelerating the assembly procedure. However, by transforming the non-affine IGABEM system in an affine way, higher memory storage is required as all the dense frequency independent parameters $\mathbf{T}_i$ and $\mathbf{q}_i$ must be stored.

1.3 Galerkin model order reduction

After approximating the original BEM system in an affine manner, a Galerkin projection can be exploited to reduce the size of the system matrix and thus tackle the high storage cost. In the Galerkin MOR, the quantity of interest $\mathbf{x}(\omega)$ is approximated by a solution $\hat{\mathbf{x}} \in \mathcal{W}$ with $\mathcal{W} := \text{span}\{\mathbf{W}\}$ which then can be expressed by a linear combination of linearly independent column vectors of $\mathbf{W} \in \mathbb{C}^{N \times l}$

$$\mathbf{x}(\omega) \approx \hat{\mathbf{x}}(\omega) = \mathbf{W} \tilde{\mathbf{x}}(\omega),$$

where $\tilde{\mathbf{x}}(\omega)$ is the reduced solution vector. Imposing the Galerkin condition $\mathbf{r}(\omega) := \mathbf{b}(\omega) - \mathbf{A}(\omega) \hat{\mathbf{x}}(\omega) \perp \mathbf{W}$, a one-sided or Galerkin projection approach is constituted to regain a square form. By employing this strategy, the following one-sided projection can be obtained

$$\mathbf{W}^H \mathbf{A}(\omega) \mathbf{W} \tilde{\mathbf{x}}(\omega) = \mathbf{W}^H \mathbf{b}(\omega),$$

where $\mathbf{W}^H$ is the Hermitian of $\mathbf{W}$. Combining the Galerkin projection in conjunction with the Chebyshev polynomial approximation of the IGABEM system, the equation can be reformulated as

$$\left(\sum_{i=0}^{\mathcal{M}} {}'c_i(\omega) \mathbf{W}^H \mathbf{T}_i \mathbf{W} \right) \tilde{\mathbf{x}}(\omega) = \sum_{i=0}^{\mathcal{M}} {}'c_i(\omega) \mathbf{W}^H \mathbf{q}_i,$$

yielding a compact form of

$$\left(\sum_{i=0}^{\mathcal{M}} 'c_i(\omega) \mathbf{T}_{i,redAKR} \right) \tilde{\mathbf{x}}(\omega) = \sum_{i=0}^{\mathcal{M}} 'c_i(\omega) \mathbf{q}_{iredAKR},$$

where $\mathbf{T}_{i,redAKR} \in \mathbb{C}^{l \times l}$ and $\mathbf{q}_{iredAKR} \in \mathbb{C}^l$ are the frequency-independent coefficients of the reduced polynomial system and $l := \text{rank}(\mathbf{W})$. Projecting each frequency-independent parameter $\mathbf{T}_i$ and $\mathbf{q}_i$ right after its assembly, reduces the size of the parameters and consequently, ensures that the memory requirements remain within the limits of the IGABEM.

1.4 Automatic Krylov subspace recycling

In order to preserve the high-fidelity property of the FOM, this work utilizes the AKR algorithm to create an initial global reduction basis $\mathbf{W}$ with $\mathcal{W} = \text{span}\{\mathbf{W}\}$. The AKR algorithm is based on Krylov subspaces recycling and offers a fast construction of a high-quality basis $\mathbf{W}$, by keeping the number of system assemblies and full solutions in the off-line phase of the MOR method to a minimum. Instead of assembling and solving $\forall \omega \in \Phi$, where Φ is a discrete set with $|\Phi| = N_{\text{tot}}$ and N_{tot} is the number of systems to be resolved, the AKR algorithm recycles Krylov subspaces only for $\omega \in \Omega_M$, where $\Omega_M := [\omega_1, \dots, \omega_L]$ is a discrete ordered set with $\Omega_M \subset \Psi$ and $|\Omega_M| \ll |\Phi|$. The residual $r(\omega)$ is used as an error estimator and evaluated during the construction of the basis $\mathbf{W}$ to guarantee a high fidelity. For a given frequency range $\Psi := [\omega_{\min}, \omega_{\max}]$ as well as a residual threshold r_{thresh} , the residual $\mathbf{r}_i(\omega)$ is computed as

$$\mathbf{r}_i(\omega) = \mathbf{r}_0(\omega) - \mathbf{A}(\omega) \tilde{\mathbf{x}}(\omega) \perp \mathbf{W}$$

where $\mathbf{r}_0(\omega)$ denotes the initial residual for $\mathbf{x}_0(\omega)$. The residual $\mathbf{r}_i(\omega)$ is computed for each iteration i and is checked to satisfy $\|\mathbf{r}(\omega)\| \leq r_{\text{thresh}} \quad \forall \omega \in \Psi$, where

$$\tilde{\mathbf{x}}_{\text{AKR}}(\omega) \in \mathbf{x}_0(\omega) + \mathcal{W}$$

with $\mathbf{x}_0: \Psi \rightarrow \mathbb{C}^N$ an initial guess of the solution. Without loss of generality, $\mathbf{x}_0 := 0$ is assumed. The Krylov subspaces are generated through an Arnoldi algorithm [20] and represented as

$$\begin{aligned} \mathcal{K}_{m(\omega_j)}(\mathbf{A}(\omega_j), \mathbf{r}_0(\omega_j)) &:= \text{span}\{\mathbf{r}_0(\omega_j), \mathbf{A}(\omega_j)\mathbf{r}_0(\omega_j), \dots, \\ &\mathbf{A}^{m-2}(\omega_j)\mathbf{r}_0(\omega_j), \mathbf{A}^{m-1}(\omega_j)\mathbf{r}_0(\omega_j)\} = \text{span}\{\mathbf{V}_{\omega_j}\}, \end{aligned}$$

where $m(\omega_j)$ with $j = 1, \dots, L$ denotes the dimension of the respective Krylov subspaces for each $\omega_j \in \Omega_M$ and $L := |\Omega_M|$. A modified Gram-Schmidt orthogonalization is performed to prevent numerical instabilities [21]. A global reduction basis $\mathbf{W}$ spans the Krylov subspaces

$$\text{span}\{\mathbf{W}\} = \mathcal{K}_{m(\omega_1)}^{\omega_1} \cup \mathcal{K}_{m(\omega_2)}^{\omega_2} \cup \dots \cup \mathcal{K}_{m(\omega_L)}^{\omega_L}.$$

The dimension $m(\omega_j)$ of the Krylov subspaces $\mathcal{K}_{m(\omega_j)}$ and Ω_M are determined iteratively through the AKR algorithm.

1.5 Efficient reduction scheme

Utilizing the single-step reduction with AKR-MOR results in only moderate reduction, as the number of Krylov subspaces to be recycled is dictated by the system's spectral properties, i.e. distribution of the eigenvalues, and the condition number. For highly resonant problems or ill-conditioned systems, the AKR-MOR does not offer substantial computational relief and the storage of the affine coefficient matrices becomes a bottleneck for the method [22].

To address this limiting factor and by taking advantage of the given reduced affine IGABEM system, a second reduction step utilizing RBM is employed to reduce the computational burden in memory and time further, enabling an efficient reduction. The RBM samples the reduced affine approximated IGABEM system and selects, following a greedy approach, the snapshots at frequency locations $\omega_{\bar{k}+1} \in \Omega_{\text{RBM}} \subset \Psi$, where Ω_{RBM} is a discrete ordered set with $|\Omega_{\text{RBM}}| \ll |\Phi|$, to build a reduction basis. A residual based error estimator for a grid $\Phi \subset \Psi$ is used for the selection of snapshots [23]. The residual error estimator is defined as

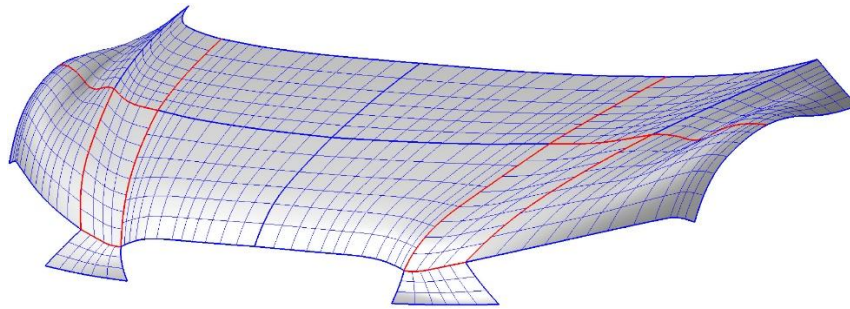
$$\mathbf{r}_{\text{RBM},\bar{k}}(\omega) := \mathbf{A}_{\text{red}}(\omega) \bar{\mathbf{V}}_{\bar{k}} \tilde{\mathbf{x}}_{2\text{Step}}(\omega) - \mathbf{b}_{\text{red}}(\omega), \forall \omega \in \Phi \subset \Psi,$$

where $\bar{\mathbf{V}}_{\bar{k}}$ denotes the reduced basis from concatenating the $\bar{k}$ snapshots. The residual $r_{\text{RBM},\bar{k}}$ is computed iteratively for all $\omega \in \Phi$. In each iteration, only the snapshot $\mathbf{x}_{\bar{k}+1}$ at the location $\omega_{\bar{k}+1}$ is chosen, which yields the worst approximation. The algorithm stops the selection until the computed residual $\mathbf{r}_{\text{RBM},\bar{k}}(\omega)$ is below a predefined threshold r_{tol} , i.e. $r_{\text{RBM}} := \|\mathbf{r}_{\text{RBM},\bar{k}}(\omega)\| \leq r_{\text{tol}}, \forall \omega \in \Phi$ and returns the reduction basis $\mathbf{V}_{\text{RBM}} := \bar{\mathbf{V}}_{\bar{k}}$ as output. During the on-line phase, the reduction basis $\mathbf{V}_{\text{RBM}}$ is used on top of the reduced affine expressed IGABEM system, enabling a fast computation and relieving the memory constraints further. The computational assessment and performance of the two-step model-order reduction method for non-affine indirect Galerkin BEM systems was investigated in [25].

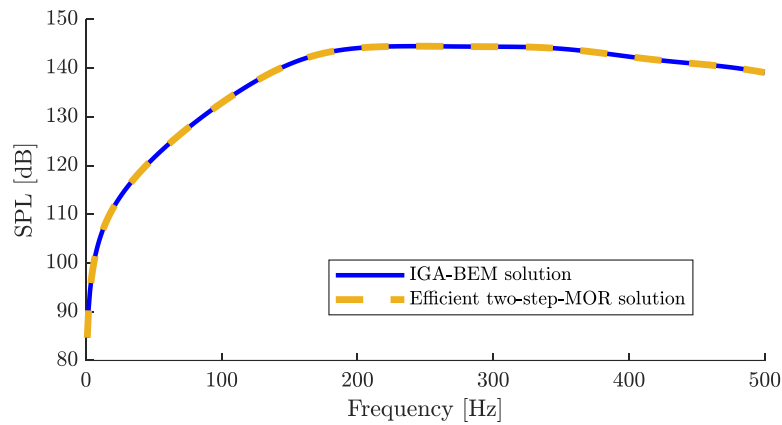
1.6 Results

The performance of the proposed two-step reduction method for acoustic IGABEM systems is investigated in terms of accuracy of the resulting ROMs and efficiency of the procedure of constructing the reduction bases. A car hood structure is studied. The dimensions of the model are roughly $1.8 \times 1.4 \times 0.4 \text{ m}$ and it consists of 14 NURBS patches with NURBS of polynomial degree 2. The structure of the car hood vibrates with a unit velocity ($\bar{v}_n = 1 \text{ m/s}$ across the entire surface) and the sound propagates through the unbounded acoustic domain. The acoustic pressure is measured at a microphone which is located at a distance of 0.6 m in height. The response is studied in a frequency range $\Psi := [1, 500] \text{ Hz}$ with a step size of 1 Hz , resulting in the sequential assembly and solution of 500 fully populated linear systems. In Figure 1(a) the non-conforming car hood is displayed and contains 1053 independent DOFs. The SPL response of the ROM produced by the two-step MOR is compared to the response by the FOM in Figure 1 (b), showing no differences, implying that the considered thresholds $r_{\text{thresh}} := 10^{-2}$ and $r_{\text{tol}} := 10^{-3}$ are sufficient. For the radiating car hood problem, a truncation order of $\mathcal{M} := 20$ was selected. By inspecting the normalized residuals of the two-step MOR method with respect to the FOM, it can be observed that the normalized residual stays below the residual threshold over the entire frequency band. The normalized residual produced by the approximated solution are displayed in Figure 2 (a). The corresponding sampling pattern for the construction $\Omega_M, \mathbf{W}$ and $\mathbf{V}_{\text{RBM}}$ are displayed in Figure 2(b). The AKR algorithm requires only 7 full system assemblies and 4 partial solutions for the construction of $\mathbf{W}$. In addition, 20 full system assemblies are employed to construct the Chebyshev coefficient matrices. The resulting ROM has a dimension of 13.

Besides inspecting the accuracy of the ROM, it is also important to analyze the computation cost of the IGABEM, the Chebyshev polynomial approximation of the IGABEM system (Cheb-IGABEM) and the ROM induced by the MOR method. As shown in Table 1, the highest computational cost for the MOR method is related to the construction of the affine coefficient matrices T_i and $\mathbf{q}_i$ which requires the full system assembly at the Chebyshev nodes. The resulting speed-up factors for the on-line stage demonstrate that the proposed method significantly accelerates both system assembly and solution compared to the FOM over the entire frequency range.

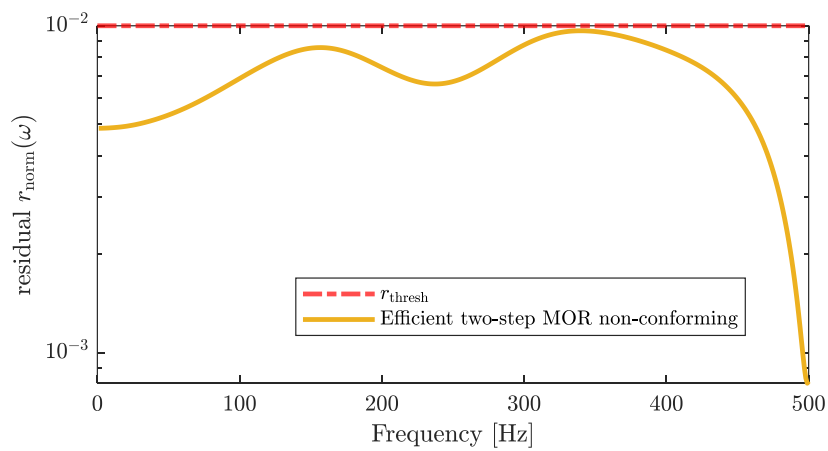


(a)

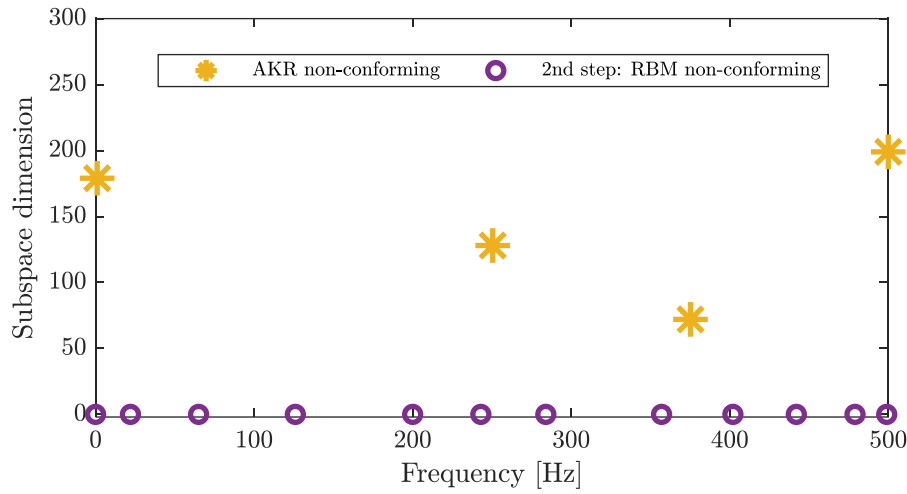


(b)

Figure 1: (a) Car hood consisting of 14 NURBS patches of which some are non-conforming (indicated with red boundaries) [8]; (b) Response at (0, 0.7, 1)



(a)



(b)

Figure 2: (a) Normalized residuals employing the proposed MOR method for the car hood problem; (b) Sampling pattern for the construction of the reduction base $\mathbf{W}$ and $\mathbf{V}_{\text{RBM}}$

Operation	IGABEM	ROM
Construction of projection basis $\mathbf{W}$	-	36 min
Construction of Chebyshev polynomial	-	2h 20 min
Total assembly time	35h 50 min	24 s
Total solution time	25 s	0.3 s

Table 1: Computational cost for a frequency sweep for the car hood problem

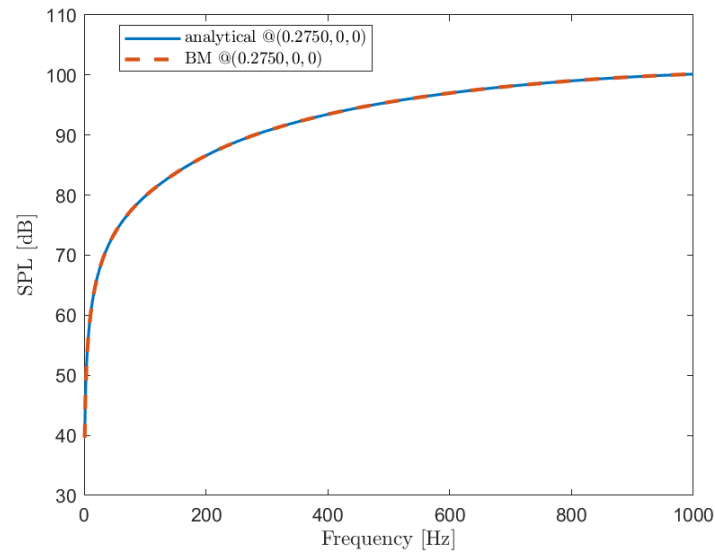
1.7 Burton-Miller formulation of the IGA indirect BEM and its application to MOR

To tackle the non-uniqueness problem associated with the exterior acoustic boundary integral formulation at characteristic frequencies, the Burton-Miller formulation was used and integrated in the MOR scheme. The Burton-Miller formulation is obtained using a linear combination of the boundary integral equation and its normal derivative which can give a unique solution at all frequencies [24] as follows:

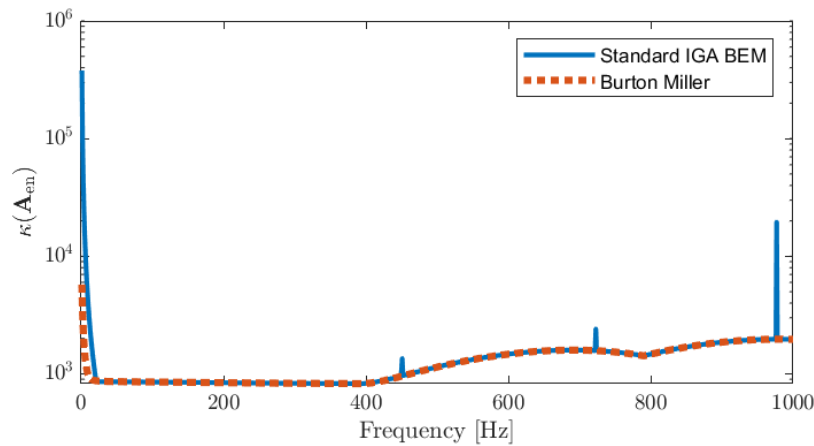
$$I_1(\mathbf{r}_f) + \eta I_2(\mathbf{r}_f) = 0,$$

where I_1, I_2, η is the boundary integral equation, its normal derivative and coupling constant, respectively. When the wavenumber $k \gg 1, \eta = \frac{j}{k}$, and $= j$ for $k < 1$.

The Burton-Miller extension of the IGA-BEM code was compared with the analytical solution of the plane wave rigid sphere in Figure 3 (a), showing that SPL curve of the Burton-Miller IGA BEM agrees with the analytical one. In Figure 3 (b), the condition number of the plane wave rigid sphere with standard IGA BEM and its Burton-Miller version was displayed to show that the Burton-Miller formulation suppresses the spurious frequencies, occurring at the internal resonance frequencies of the sphere.



(a)



(b)

Figure 3: (a) Response at $(0.2750, 0, 0)$ between analytical and Burton-Miller IGA indirect Galerkin BEM, (b) Condition number of standard IGA indirect Galerkin BEM and Burton-Miller IGA indirect Galerkin BEM

The two-step MOR approach utilizing the Burton-Miller version of the IGABEM for the plane sphere case with 486 DOFs show that the normalized residual is in general below the residual threshold except for the frequency range of 180 Hz-210 Hz (see Figure 4 (a)). The peak at the lower frequencies can be explained by the bisection strategy of the two-step MOR method which is displayed in Figure 4 (b).

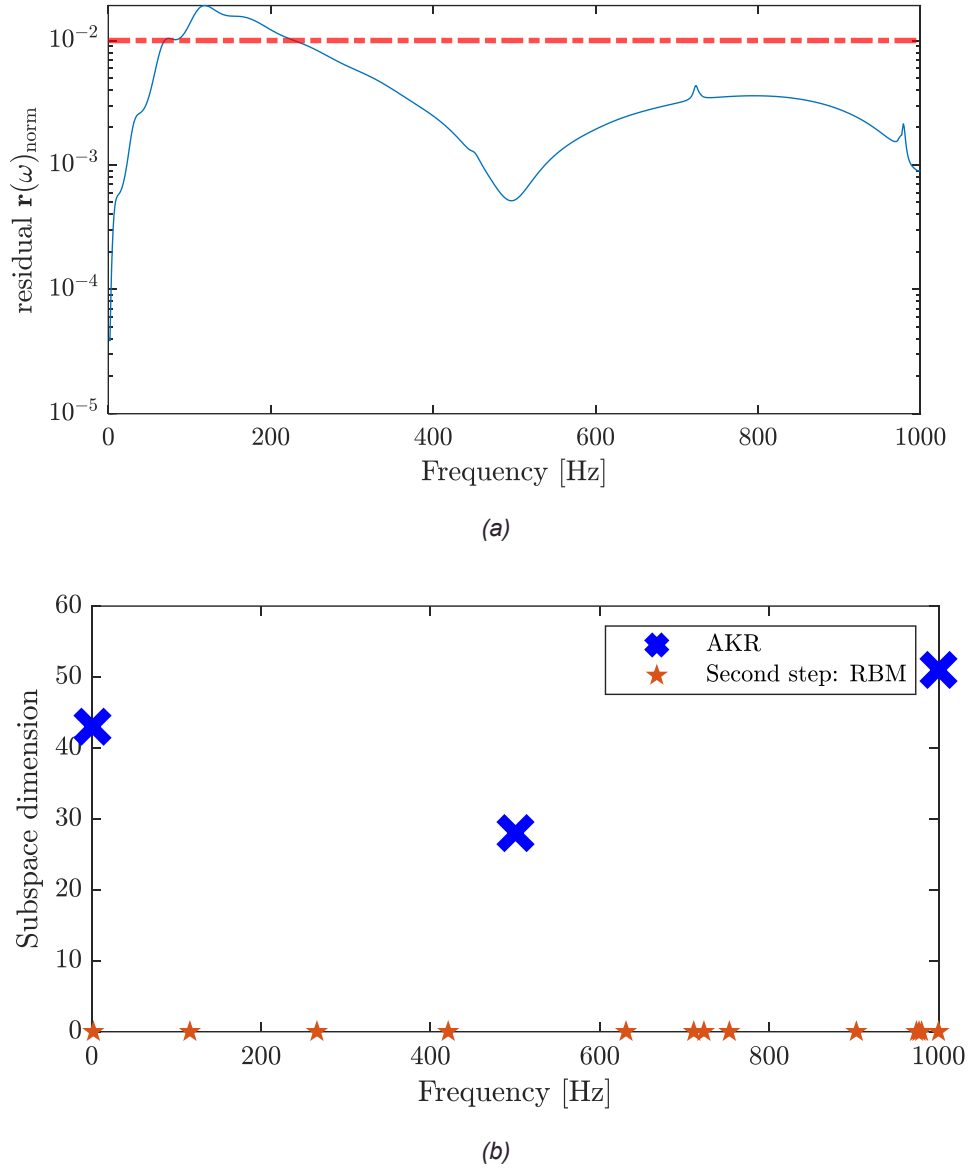


Figure 4: (a) Normalized residual between the FOM using Burton-Miller IGABEM and its ROM, (b) Condition number of standard IGA indirect Galerkin BEM and Burton-Miller IGA indirect Galerkin BEM

1.8 Model order reduction for IGABEM

Enabling multi-frequency solution of IGABEM through MOR unlock efficient broadband shape optimization, where a large number of frequency-domain simulations are required to efficiently explore the design space. Specifically, MOR allows to cut down the cost per design iteration, while exact geometric representation offered by IGABEM allows design exploration without explicitly remeshing the geometry. Despite the computational gains offered by the ROM, the underlying IGABEM system matrix is often ill-conditioned, which can affect the robustness of sensitivity-based optimization methods that rely on accurate gradient information. In particular, numerical sensitivity approximations such as finite-difference method may suffer from instability and

numerical noise in the presence of ill-conditioned systems. For this reason, gradient-free optimization have been explored in conjunction with MOR schemes for BEM.

1.9 Particle swarm optimization for acoustic broadband frequencies using ROMs

The broadband acoustic behaviour of the car hood is investigated to illustrate the capabilities of the MOR and gradient free optimization using particle swarm optimization for practical shape optimization applications. The hood is represented as a non-conforming multi-patch NURBS surface with 1053 independent DOFs. The acoustic excitation is modelled via a monopole source of unit amplitude positioned beneath the hood at $[0, 0.6, -1]$ m, approximately 1 m below the geometric center. This source configuration simulates the radiation of the engine compartment while leaving the upper surface free to scatter sound into the surrounding space. The objective of the optimization is to minimize the broadband scattered sound pressure average across all microphone positions and over the frequency range of $\Psi: = [1, 500]$ Hz. A hemispherical array of 50 microphones is positioned above the car hood to measure the scattered sound field in the exterior domain, as shown in Figure 5. The hemisphere has a radius of 1 m, with its center aligned to the geometric center of the car hood. This configuration captures the directivity of the scattered field and provides a measure of overall noise radiation. A total of 12 design variables is selected from the control mesh of the car hood, where each design variable is allowed to move within a range of ± 0.1 m, representing a complete redesign of the hood geometry rather than a local fine-tuning. The selected DVs are illustrated in Figure 6. The PSO algorithm is employed with 12 particles and a maximum of 100 iterations. Convergence is declared when the improvement of the global best objective value over the last five iterations falls below 10^{-3} . For the considered setup, the optimization algorithm converged after 33 iterations, as seen in Figure 7. The convergence history demonstrates a steady decrease of the broadband objective.

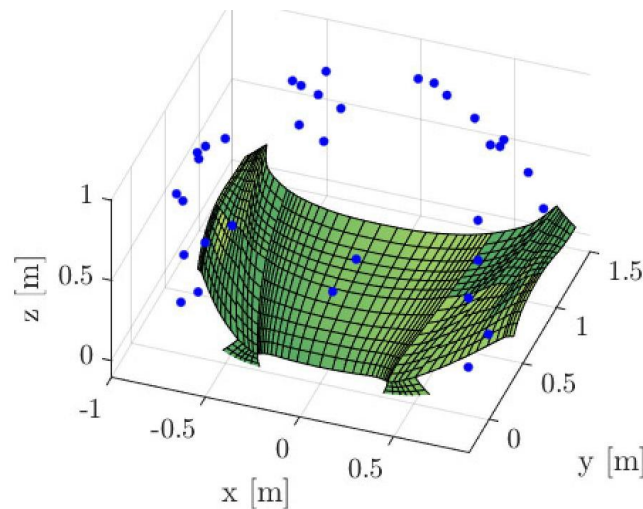


Figure 5: Initial car hood shape with microphone arrays (marked as blue dots)

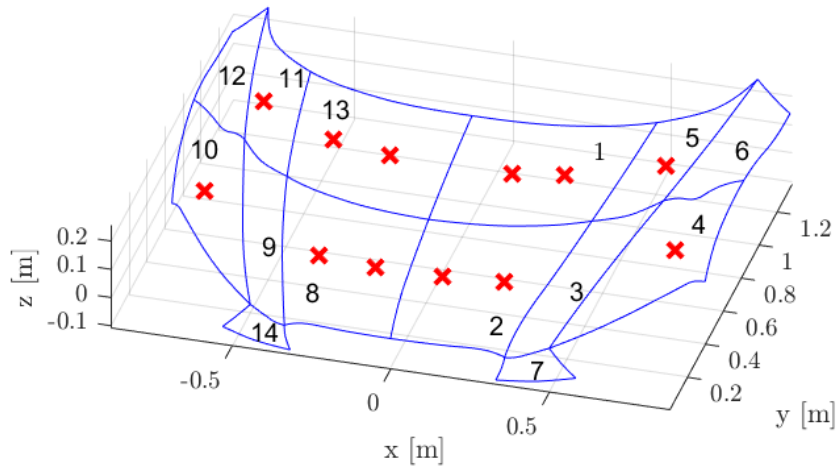


Figure 6: Patch indices with design variable location (marked as red crosses) for shape optimization

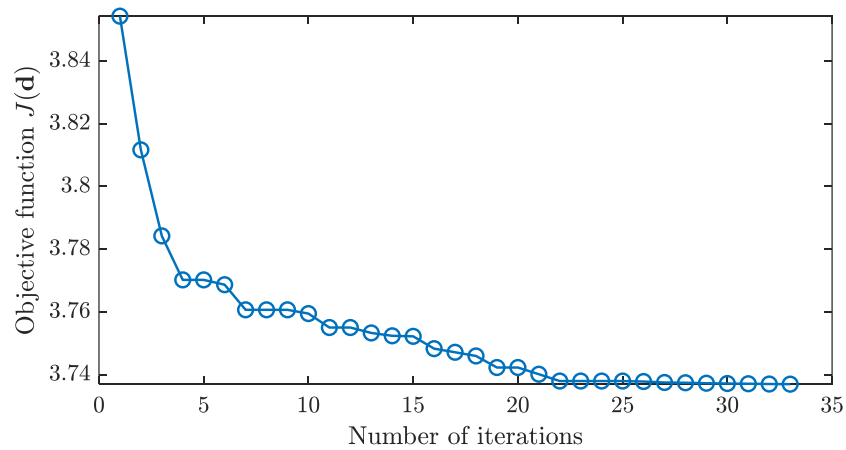


Figure 7: Convergence history of the particle swarm optimization MOR framework for broadband acoustic shape optimization of the car hood.

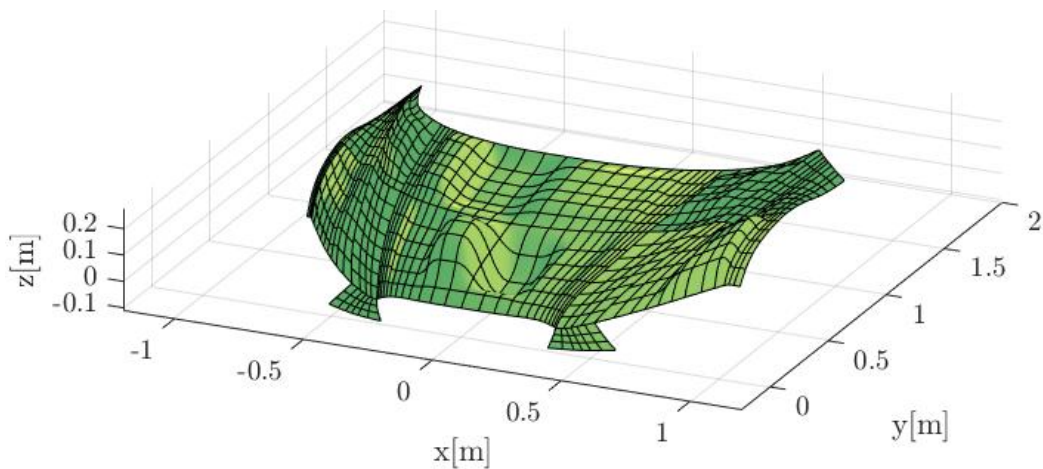


Figure 8: Optimized shape of the car hood

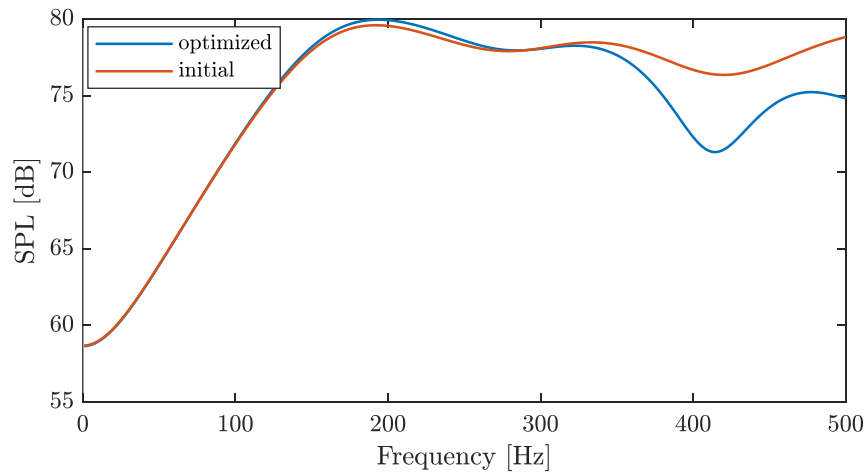


Figure 9: SPL spectrum at microphone located (0.6, -0.06, 0.7) m

The optimized car hood geometry (Figure 8) shows pronounced global modifications, including both local dents and protrusions, corresponding to the large admissible control-point displacements of ± 0.1 m. These deformations reflect a complete redesign of the hood rather than a minor shape refinement. Despite the exaggerated geometric freedom, the resulting shape changes remain smooth due to the high-continuity NURBS representation and the topological ring-based smoothing, illustrating the robustness of the proposed IGA MOR framework in handling large-amplitude, broadband shape updates. Although the broadband average SPL reduction across all microphones is modest (≈ 0.2 dB), more pronounced localized reductions are observed at specific microphone locations on the hemispherical array. In particular, a microphone located at (0.6, -0.06, 0.7) m exhibits the largest broadband improvements, as illustrated in Figure 9. These results indicate that the proposed MOR framework is capable of identifying shape modifications that lead to localized acoustic improvements, even when the global average reduction remains limited. To highlight the computational gains of the IGA MOR framework for shape optimization, a summary of the optimization parameters, and results is given in Table 2, illustrating that a smaller number of full system assemblies and solutions is required compared to the FOM approach.

The results presented in this section concerning broadband acoustic shape optimization using PSO and reduced-order IGABEM models are reported in [26].

Parameter	Value
Number of design variables	12
Frequency range	1-500 Hz
Number of particles	12
Maximum iterations	100
Convergence iteration	33
Total full size system evaluations	198,000 (FOM) / 10,692 (MOR) (5.4%)

Overall speed-up factor using ROM	18.5
--	------

Table 2 Summary of the optimization setup and outcome for the non-conforming car hood problem

2 CONCLUSIONS

In this research, an efficient two-step reduction method was proposed for acoustic IGA indirect Galerkin BEM systems. The IGA-BEM is first verified by an analytical solution of the plane wave scattering problem by a rigid sphere. Afterwards, based on the FOM of the IGA-BEM, the presented technique uses Chebyshev polynomials to express the IGA-BEM system in an affine expression, and a Galerkin projection is deployed for the order reduction of the resulting affine system. The reduction basis is created by leveraging the Krylov subspaces recycling of [18] and by using RBM as a second reduction step to compress the size further. The MOR method is analyzed in terms of accuracy and computational cost, showing that a substantial speed up in the on-line phase is offered.

Furthermore, the Burton-Miller formulation was incorporated into the IGABEM and combined with the proposed two-step MOR approach. The resulting formulation was validated for the plane wave scattering problem by a rigid sphere, demonstrating that the spurious frequencies associated with the boundary integral formulation can be avoided while maintaining the accuracy of the reduced-order solution. Finally, the applicability of the proposed MOR approach to broadband acoustic shape optimization was investigated by combining the reduced-order IGABEM model with a gradient-free optimizer. The results demonstrate that the MOR method can substantially reduce the computational cost of repeated frequency-analyses.

3 REFERENCES

- [1] T. Hughes, J. Cottrell, and Y. Bazilevs, Isogeometric analysis: Cad, finite elements, nurbs, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, vol. 194, no. 39, pp. 4135-4195, 2005.
- [2] W. Heylen, S. Lammens, and P. Sas, *An Introduction to NURBS with Historical Perspective*. San Diego, CA: Academic Press, 2001.
- [3] J. Kiendl, K.-U. Bletzinger, J. Linhard and R. Wüchner, Isogeometric shell analysis with kirchhoff-love elements. *Computer Methods in Applied Mechanics and Engineering*, vol. 198, no. 49, pp. 3902-3914, 2009.
- [4] F. Greco, A. Rosolen, L. Coox, and W. Desmet, Contact mechanics with maximum-entropy meshfree approximants blended with isogeometric analysis on the boundary. *Computers Structures* vol. 182, pp. 165-175, 2017.
- [5] P. N. Nielsen, A. R. Gersborg, J. Gravesen, and N. L. Pedersen, Discretizations in isogeometric analysis of navier-stokes flow. *Computer Methods in Applied Mechanics and Engineering*, vol. 200, no. 45, pp. 3242-3253, 2011
- [6] Bazilevs, M.-C. Hsu, and M. Scott, “Isogeometric fluid-structure interaction analysis with emphasis on non-matching discretizations, and with application to wind turbines,” *Computer Methods in Applied Mechanics and Engineering*, vol. 249-252, pp. 28-41, 2012.
- [7] R. Simpson, M. Scott, M. Taus, D. Thomas, and H. Lian, Acoustic isogeometric boundary element analysis. *Computer Methods in Applied Mechanics and Engineering*, vol. 269, pp. 265-290, 2014.
- [8] L. Coox, O. Atak, D. Vandepitte, and W. Desmet, An isogeometric indirect boundary element method for solving acoustic problems in open-boundary domains. *Computer Methods in Applied Mechanics and Engineering*, vol. 316, pp. 186-208, 2017, special Issue on Isogeometric Analysis Progress and Challenges.
- [9] E. O. Inci, L. Coox, O. Atak, E. Deckers, and W. Desmet, Applications of an isogeometric indirect boundary element method and the importance of accurate geometrical representation in acoustic problems. *Engineering Analysis with Boundary Elements*, vol. 110, pp. 124-136, 2020.
- [10] D. Panagiotopoulos, W. Desmet, and E. Deckers, Parametric model order reduction for acoustic boundary element method systems through a multiparameter krylov subspaces recycling strategy. *International Journal for Numerical Methods in Engineering*, vol. 123, no. 22, pp. 5546-5569, 2022.
- [11] Liang, H. Lee, S. Lim, W. Lin, K. Lee, and C. Wu, Proper orthogonal decomposition and its applications—part i: Theory. *Journal of Sound and Vibration*, vol. 252, no. 3, pp. 527-544, 2002.
- [12] S. K. Baydoun, M. Voigt, C. Jelic, and S. Marburg, A greedy reduced basis scheme for multifrequency solution of structural acoustic systems. *International Journal for Numerical Methods in Engineering*, vol. 121, no. 2, pp. 187-200, 2020.
- [13] Z. Bai, Krylov subspace techniques for reduced-order modeling of large-scale dynamical systems. *Applied Numerical Mathematics*, vol. 43, no. 1, pp. 9-44, 2002.
- [14] A. van de Walle, F. Naets, E. Deckers, and W. Desmet, Stability-preserving model order reduction for time-domain simulation of vibro-acoustic fe models. *International Journal for Numerical Methods in Engineering*, vol. 109, no. 6, pp. 889-912, 2017.
- [15] Y. Cai, S. van Ophem, W. Desmet, and E. Deckers, Model order reduction of time- domain vibro-acoustic finite element simulations with poroelastic

- materials. *Computer Methods in Applied Mechanics and Engineering*, vol. 426, p. 116980, 2024
- [16] D. Panagiotopoulos, E. Deckers, and W. Desmet, Krylov subspaces recycling based model order reduction for acoustic bem systems and an error estimator. *Computer Methods in Applied Mechanics and Engineering*, vol. 359, p. 112755, 2020.
- [17] D. Panagiotopoulos, W. Desmet, and E. Deckers, An automatic krylov subspaces recycling technique for the construction of a global solution basis of non-affine parametric linear systems. *Computer Methods in Applied Mechanics and Engineering*, vol. 373, p. 113510, 2021.
- [18] T. Wu, *Boundary Element Acoustics, Fundamentals and Computer Codes*. WIT Press, 2000.
- [19] T. Liang, J. Wang, J. Xiao, L. Wen, Coupled be-fe vibroacoustic modal analysis and frequency sweep using a generalized resolvent sampling method, *Computer Methods in Applied Mechanics and Engineering*, vol. 345, p. 5546-5569, 2019
- [20] W. E. Arnoldi, The principle of minimized iterations in the solution of the matrix eigenvalue problem, *Quarterly of Applied Mathematics* 9 (1), p.17-29 (1951)
- [21] A. Björck, Numerics of gram-schmidt orthogonalization, *Linear Algebra and its Applications* 197-198, p. 297-316 (1994).
- [22] D. Panagiotopoulos, E. Deckers, W. Desmet, A two-step reduction method for acoustic BEM systems, *Forum Acusticum*, Lyon, France, p. 2291-2298 (2020).
- [23] S.K. Baydoun, M. Voigt, C. Jelic, S. Marburg, A greedy reduced basis scheme for multifrequency solution of structural acoustic systems, *International Journal for Numerical*
- [24] A. J. Burton, GF Miller, The application of integral equation methods to the numerical solution of some exterior boundary-value problems, *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences* (1971): 201-210
- [25] P. Le, D. Panagiotopoulos, E. Deckers, A non-intrusive efficient two-step Model reduction method for Large-Scale thin-walled acoustic indirect BEM systems. *International Journal for Numerical Methods in Engineering*. 126 (20). Art. No. e70171
- [26] P. Le, D. Panagiotopoulos, E. O. Inci, E. Deckers, Model order reduction approach towards efficient shape optimization for non-conforming multi-patch isogeometric boundary element method, *Engineering Analysis with Boundary Elements*, 191.